Control Systems(18EC43)

By

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Text Book:

- J.Nagarath and M.Gopal,— ControlSystemsEngineering‖, NewAge International (P) Limited, Publishers,

Fifth edition-2005, ISBN: 81-224-2008-7.

Reference Books:

- ModernControlEngineering,‖K.Ogata,PearsonEducationAsia/PHI,4thEdition, 2002. ISBN 978-81-203-4010-7.
- AutomaticControl Systems‖,BenjaminC.Kuo,JohnWiley India Pvt. Ltd., 8thEdition, 2008.
- JosephJDistefano III et al.,Schaum‘s Outlines, TMH, 2nd Edition2007

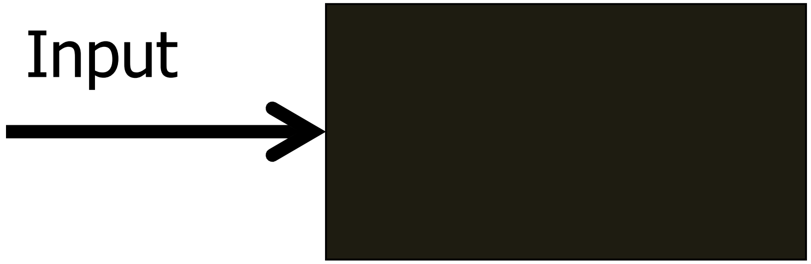
Introduction to Control System

COURSE OUTCOMES(CO’s):

At the end of the course, the students will be able to

- Develop the mathematical model of mechanical and electrical systems.
- Evaluate the transfer function for a given system using block diagram reduction techniques and signal flow graph method.
- Determine the time domain specification for first and second order systems and analyse the working of PID Controller.
- Determine the stability of a system using Routh-Hurwitz criterion and Root-locus technique.
- Determine the stability of a system in the frequency domain using Nyquist, Polar Plots and bode plots and also analyse the system using state variable and state models

Input

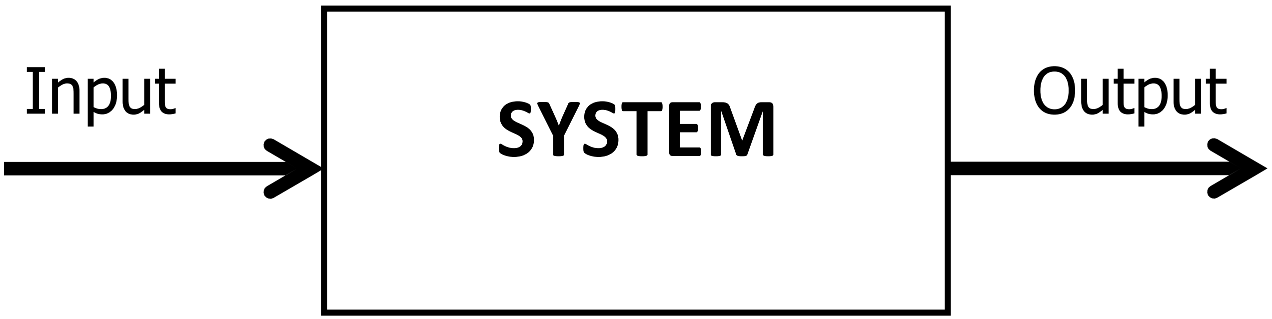


- The stimulus or excitation applied to a control system from an external source in order to produce the output is called input

Output

- The actual response obtained from a system is called output.

“System”



- A system is an arrangement of or a combination of different physical components connected or related in such a manner so as to form an entire unit to attain a certain objective.

Control

- It means to regulate, direct or command a system so that the desired objective is attained

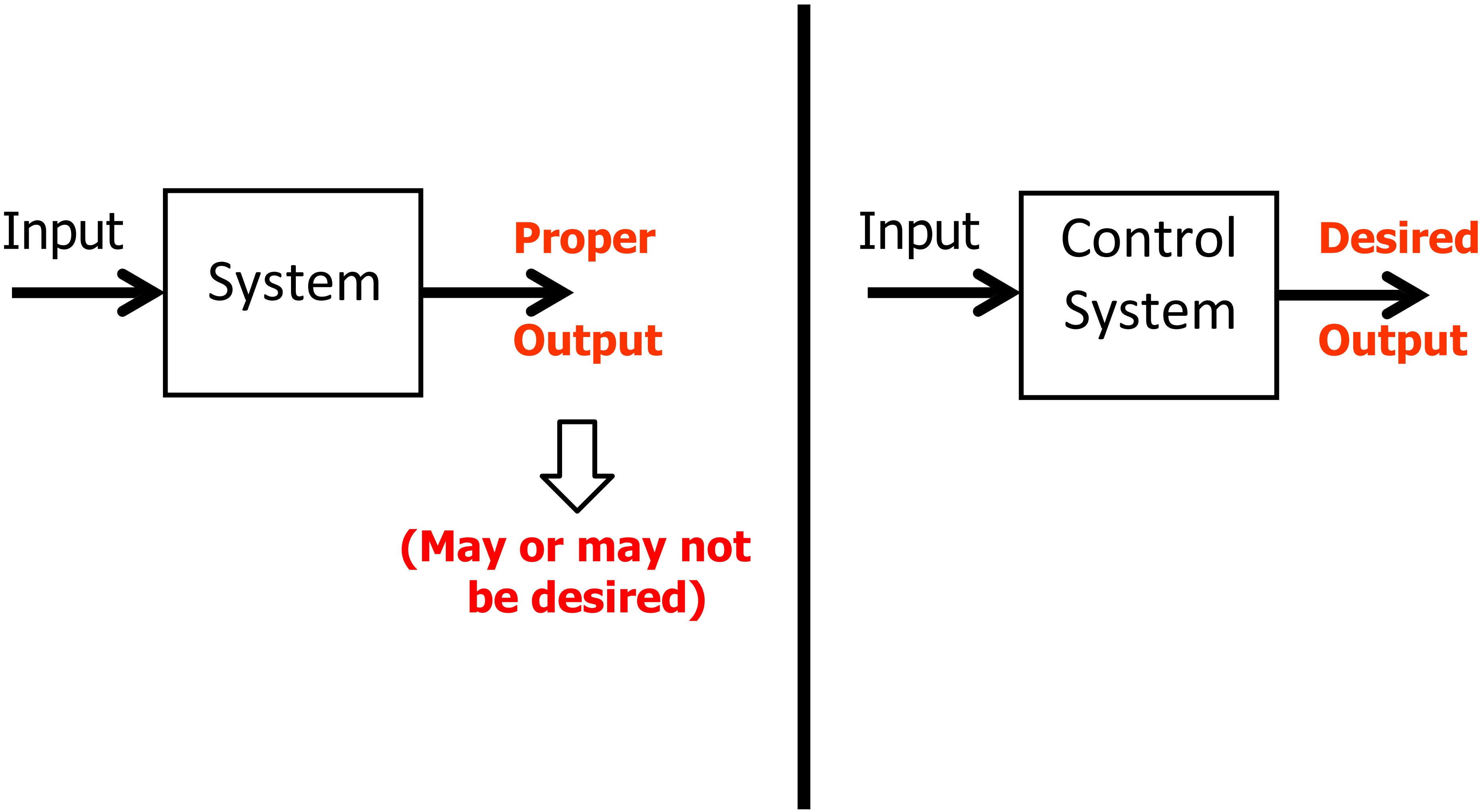
Combining above definitions

System + Control = Control System

Control System

- It is an arrangement of different physical elements connected in such a manner so as to regulate, direct or command itself to achieve a certain objective.

Difference between System and Control System



Difference between System and Control System

An example : Fan

Input { 230V/50Hz → AC Supply } → [Fan (System)] → { Air Flow } Output

A Fan: Can't Say System

- A Fan without blades cannot be a "SYSTEM"

  Because it cannot provide a desired/proper output

  i.e. airflow

Input

230V/50Hz

AC Supply

Output

No Airflow

(No Proper/ Desired Output)

A Fan: Can be a System

- A Fan with blades but without regulator can be a "SYSTEM" Because it can provide a **proper output** i.e. airflow
- But it cannot be a "Control System" Because it cannot provide desired output i.e. controlled airflow

Input

230V/50Hz

AC Supply

Output

Airflow

(Proper Output)

A Fan: Can be a Control System

- A Fan with blades and with regulator can be a "CONTROL SYSTEM" Because it can provide a **<u>Desired output.</u>**

  i.e. Controlled airflow

Input

230V/50Hz

AC Supply

Control Element

Output

Controlled Airflow

(Desired Output)

Classification of Control System

Classification of Control System

(Depending on control action)

- **Open Loop Control System**
- **Closed Loop Control System**

Open Loop Control System

Definition:

"A system in which the control action is totally independent of the output of the system is called as open loop system"

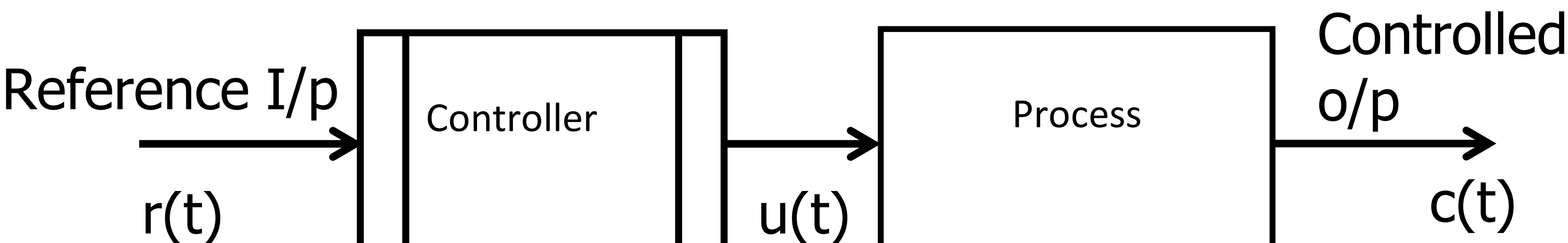


Fig. Block Diagram of Open loop Control System

OLCS Examples

- **<u>Electric hand drier</u>** – Hot air (output) comes out as long as you keep your hand under the machine, irrespective of how much your hand is dried.

OLCS Examples

- **<u>Automatic washing machine</u>**
  - – This machine runs according to the pre-set time irrespective of washing is completed or not.

OLCS Examples

- **<u>Bread toaster</u>** - This machine runs as per adjusted time irrespective of toasting is completed or not.

OLCS Examples

- **Automatic tea/coffee Vending Machine** – These machines also function for pre adjusted time only.

OLCS Examples

- **Light switch** – lamps glow whenever light switch is on irrespective of light is required or not.

- **Volume on stereo system** – Volume is adjusted manually irrespective of output volume level.

Advantages of OLCS

- Simple in construction and design.
- Economical.
- Easy to maintain.
- Generally stable.
- Convenient to use.

Disadvantages of OLCS

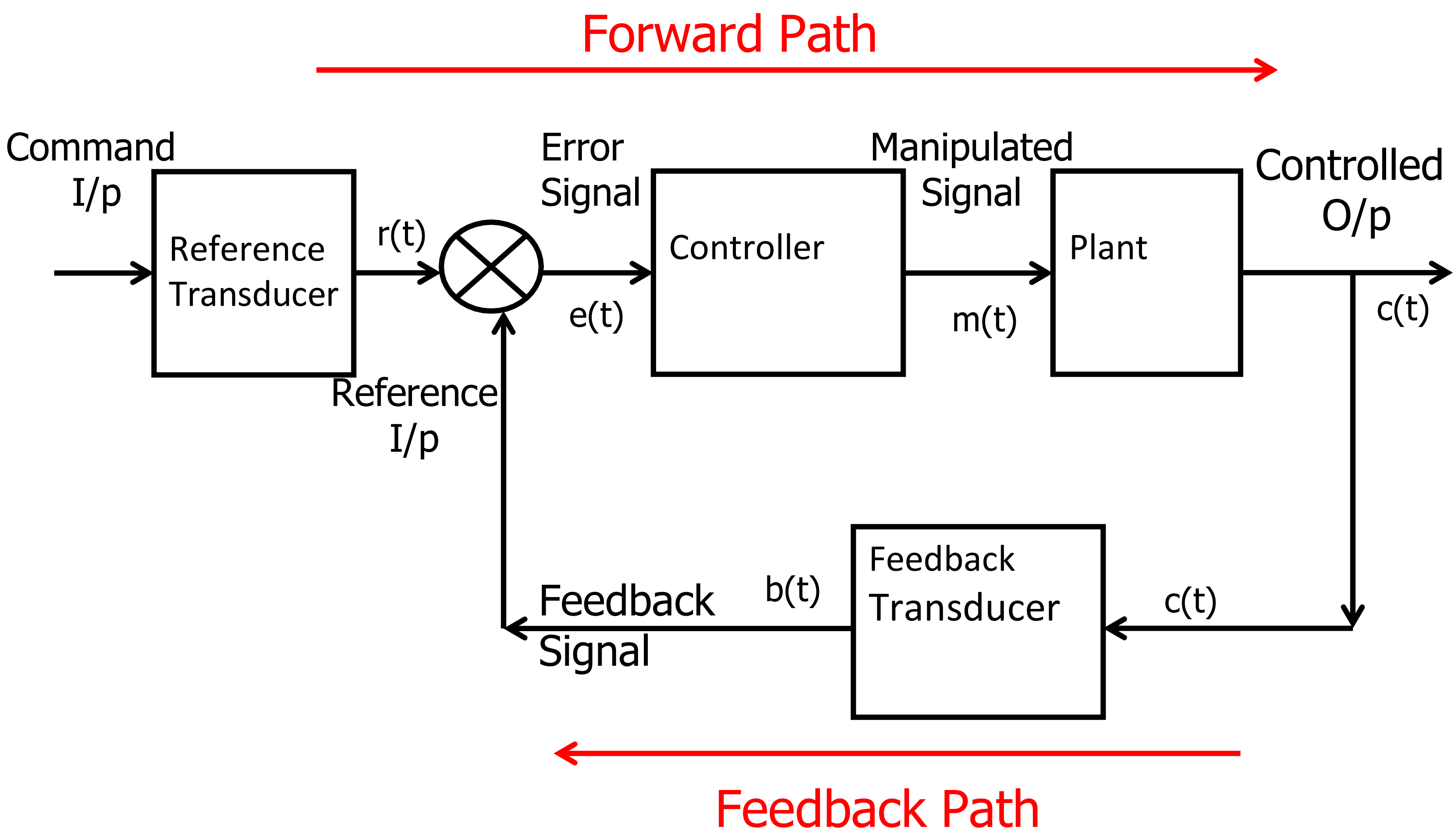
- They are inaccurate
- They are unreliable
- Any change in output cannot be corrected automatically.

Closed Loop System

Definition:

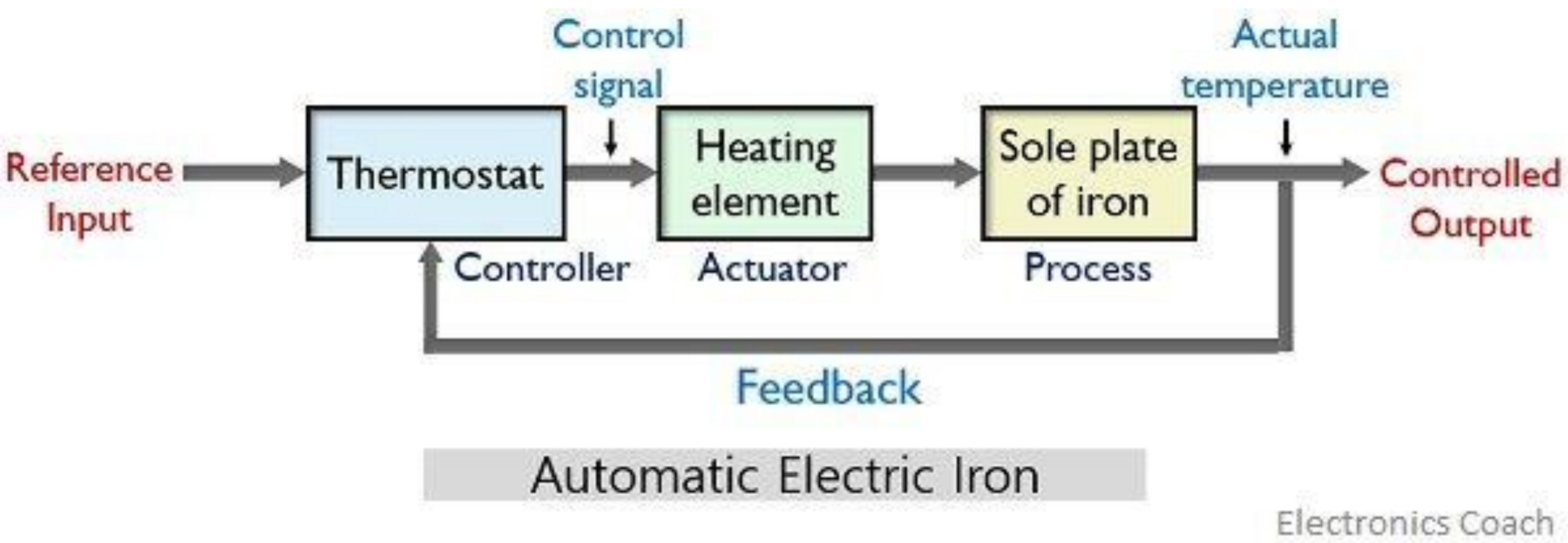
"A system in which the control action is somehow dependent on the output is called as closed loop system"

Block Diagram of CLCS



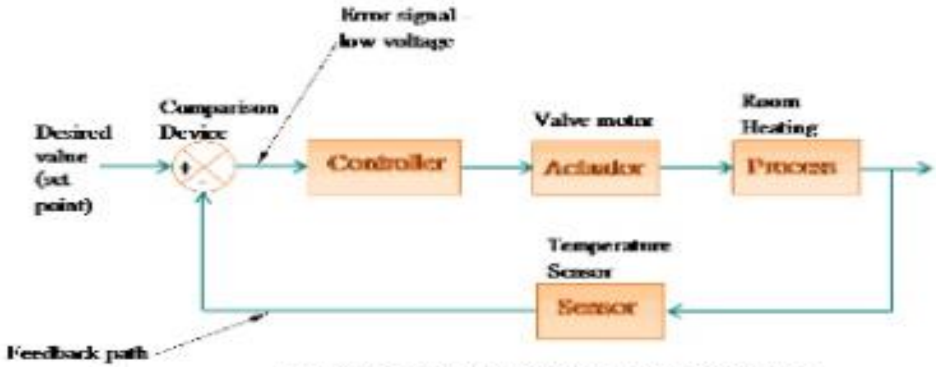
CLCS Examples

- **<u>Automatic Electric Iron</u>**- Heating elements are controlled by output temperature of the iron.



CLCS Examples

2.Closed -loop heating system control



HEATING SYSTEM CONTROL LOOP DIAGRAM

- The temperature sensor is installed in the room to be controlled and sends a signal back along the feedback path to the comparison device incorporated in the controller.
- The comparison device compares the value of temperature at the sensor to that of the desired value or set point on the controller.

Advantages of CLCS

- Closed loop control systems are more accurate even in the presence of non-linearity.
- Highly accurate as any error arising is corrected due to presence of feedback signal.
- Bandwidth range is large.
- Facilitates automation.
- The sensitivity of system may be made small to make system more stable.
- This system is less affected by noise.

Disadvantages of CLCS

- They are costlier.
- They are complicated to design.
- Required more maintenance.
- Feedback leads to oscillatory response.
- Overall gain is reduced due to presence of feedback.
- Stability is the major problem and more care is needed to design a stable closed loop system.

Open Loop Control System

1. The open loop systems are simple & economical.
2. They consume less power.
3. The OL systems are easier to construct because of less number of components required.
4. The open loop systems are inaccurate & unreliable

Closed Loop Control System

1. The closed loop systems are complex and costlier
2. They consume more power.
3. The CL systems are not easy to construct because of more number of components required.
4. The closed loop systems are accurate & more reliable.

Open Loop Control System	Closed Loop Control System
5. Stability is not a major problem in OL control systems. Generally OL systems are stable.	5. Stability is a major problem in closed loop systems & more care is needed to design a stable closed loop system.
6. Small bandwidth.	6. Large bandwidth.
7. Feedback element is absent.	7. Feedback element is present.
8. Output measurement is not necessary.	8. Output measurement is necessary.

Open Loop Control System

9. The changes in the output due to external disturbances are not corrected automatically. So they are more sensitive to noise and other disturbances.

10. Examples:

    Coffee Maker,

    Automatic Toaster,

    Hand Drier.

Closed Loop Control System

9. The changes in the output due to external disturbances are corrected automatically. So they are less sensitive to noise and other disturbances.

10. Examples:Guided

    Missile,

    refrigerator, AC

Classification of Control System

Classification of Control System

Linear Control System

Non-linear Control System

Linear Control System

- When an input X1 produces an output Y1 & an input X2 produces an output Y2, then any combination $\alpha X1 + \beta X2$ should produce an output $\alpha Y1 + \beta Y2$. In such case system is linear. Therefore, linear systems are those where the principles of superposition and proportionality are obeyed.

Non-linear Control System

- Non-linear systems do not obey law of superposition.
- The stability of non-linear systems depends on root location as well as initial conditions & type of input.
- Non-linear systems exhibit self sustained oscillations of fixed frequency.

Difference Between Linear & Non-linear System

Linear System	Non-linear System
1. Obey superposition.	1. Do not obey superposition
2. Can be analyzed by standard test signals	2. Cannot be analyzed by standard test signals
3. Stability depends only on root location	3. Stability depends on root locations, initial conditions & type of input
4. Do not exhibit limit cycles	4. Exhibits limit cycles
5. Do not exhibit hysteresis/ jump resonance	5. Exhibits hysteresis/ jump resonance
6. Can be analyzed by Laplace transform, z- transform	6. Cannot be analyzed by Laplace transform, z- transform

Classification of Control System

Classification of Control System

- Time Varying Control System
- Time Invariant Control System

Time varying/In-varying Control System

- Systems whose parameters vary with time are called time varying control systems.
- When parameters do not vary with time are called Time Invariant control systems.

Time varying/In-varying Control System

- The mass of missile/rocket reduces as fuel is burnt and hence the parameter mass is time varying and the control system is time varying type.

Laplace Transform

- Developed mathematics in astronomy, physics, and statistics

- Began work in calculus which led to the Laplace Transform

- Focused later on celestial mechanics

- One of the first scientists to suggest the existence of black holes

The French Newton
Pierre-Simon Laplace

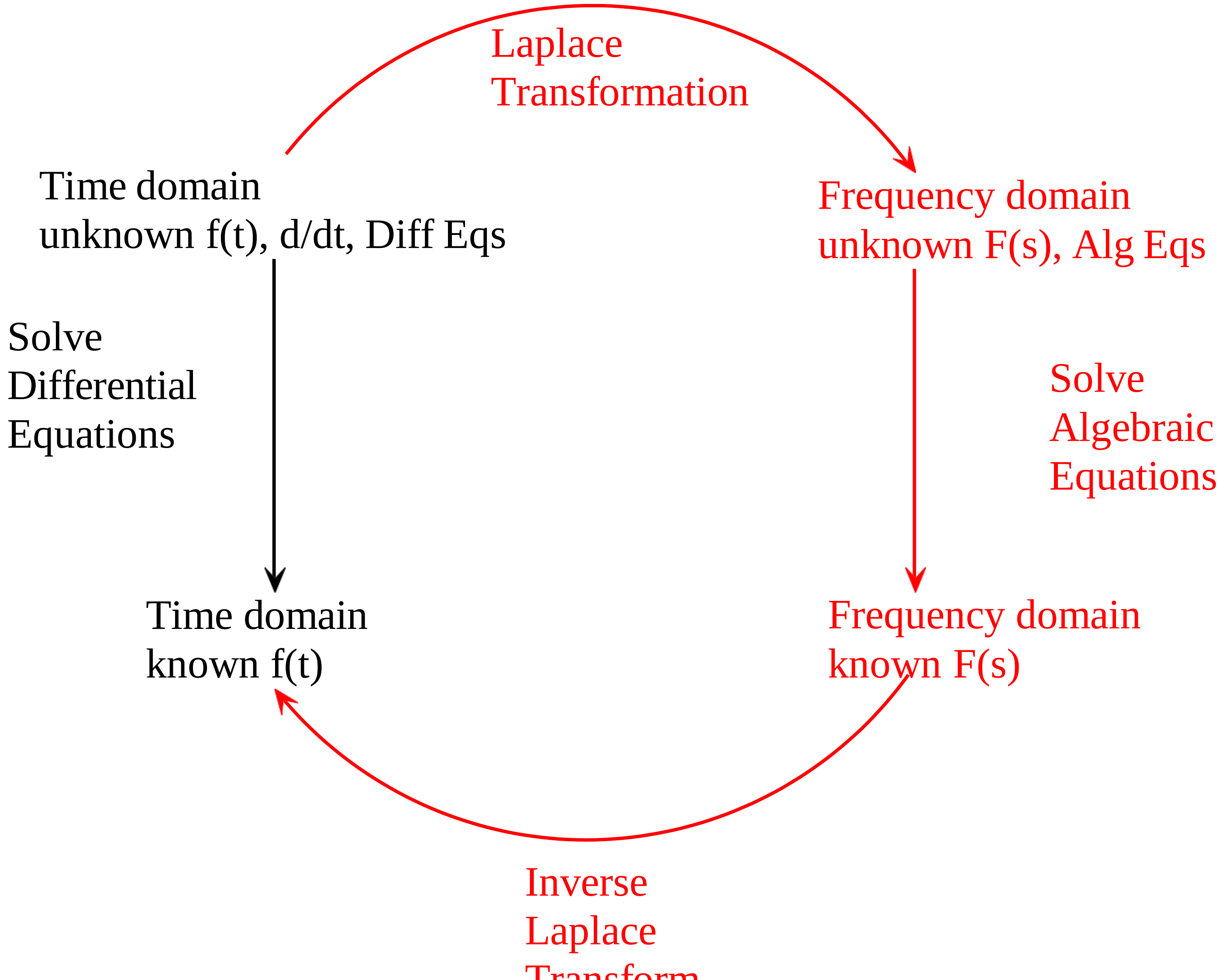
Laplace Transform

- To evaluate the performance of an automatic control system commonly used mathematical tool is **"Laplace Transform"**

- Laplace transform converts the differential equation into an algebraic equation in 's'.

- **Laplace transform exist for almost all signals of practical interest.**

Why Laplace Transform?



Advantages of Laplace Transform

- Solution of intego differential equation of time systems can be easily obtained.
- Initial conditions are automatically incorporated.
- Laplace transform provides an easy & effective solution of many problems arising in automatic control systems.
- Laplace transform allows the use of graphical techniques, for predicting the system performance.

Laplace Transform- Definition

The Laplace transform of a function, *f*(*t*), is defined as

$$F(s) = \mathcal{L}\left[f(t)\right] = \int_0^\infty f\left(t\right)e^{-st}dt \qquad (1\text{-}1)$$

where *F*(*s*) is the symbol for the Laplace transform, L is the Laplace transform operator, and *f*(*t*) is some function of time, *t*.

Note: The L operator transforms a time domain function *f*(*t*) into an *s* domain function, *F*(*s*). *s* is a *complex variable*:
$s = a + bj, \quad j \triangleq \sqrt{-1}$

Standard Laplace Transform

$f(t)$	$F(s) = L[f(t)]$
1 or $u(t)$	$\frac{1}{s}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-\alpha t}\sin\omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos\omega t$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$e^{-\alpha t}t^n$	$\frac{n!}{(s+\alpha)^{n+1}}$
$\delta(t)$	1

*Use when roots are complex.

Inverse Laplace Transform

By definition, the inverse Laplace transform operator, L^{-1}, converts an *s*-domain function back to the corresponding time domain function:

$$f(t) = L^{-1}\left[F(s)\right]$$

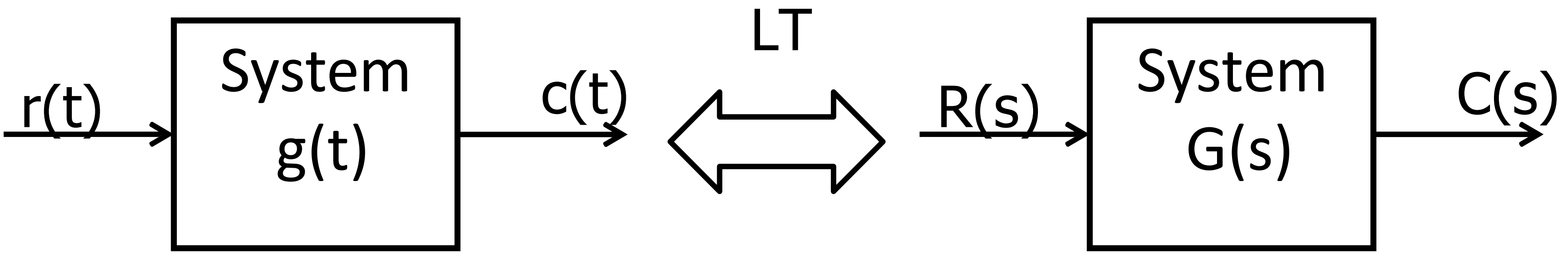
Module I – Introduction to Control System

- Introduction to Control systems (4 Marks)
  - Control System – Definition and Practical Examples
  - Classification of Control System : Open Loop and Closed Loop Systems – Definitions, Block diagrams, practical examples, and Comparison, Linear and Non-linear Control System, Time Varying and Time In-varying Systems
  - Servo System : Definition, Block Diagram, Classification (AC and DC Servo System), Block diagram of DC Servo System.
- Laplace Transform and Transfer Function (4 Marks)
  - Laplace Transform : Signifiance in Control System
  - **Transfer Function : Definition, Derivation of transfer functions for Closed loop Control System and Open Loop Control System, Differential Equations and transfer functions of RC and RLC Circuit**
- **Block Diagram Algebra (8 Marks)**
  - Order of a System : Definition, 0,1,2 order system Standard equation, Practical Examples
  - Block Diagram Reduction Technique: Need, Reduction Rules, Problems

Transfer Function

- The relationship between input & output of a system is given by the transfer function.
- **Definition:** The ratio of Laplace transform of the output to the Laplace transform of the input under the assumption of zero initial conditions is defined as **"Transfer Function"**.

Transfer Function



For the system shown,

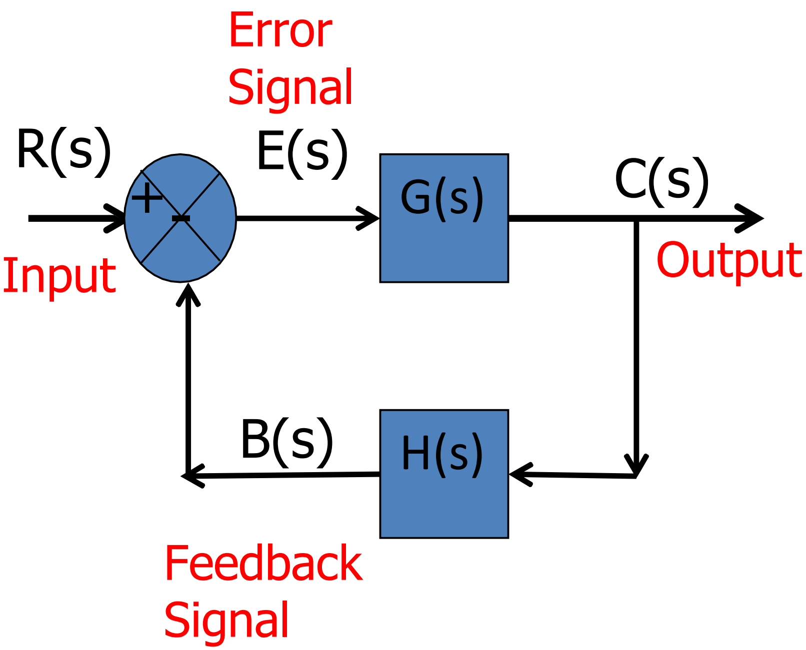
c(t)= output
r(t)= input
g(t)= System function

L{c(t)}= C(s)
L{r(t)}= R(s)
L{g(t)}= G(s)

Therefore transfer function G(s) for above system is given by,

$$G(s) = \frac{\text{Laplace of output}}{\text{Laplace of input}} = \frac{C(s)}{R(s)}$$

Transfer Function of closed loop system



Error signal is given by;

$$E(s) = R(s) - B(s) ------(1)$$

$$\therefore R(s) = E(s) + B(s)$$

Gain of feedback network is given by;

$$H(s) = \frac{B(s)}{C(s)}$$

$$B(s) = H(s).C(s) ------(2)$$

Gain for CL system is given by;

$$G(s) = \frac{C(s)}{E(s)}$$

$$\therefore C(s) = G(s).E(s) ------(3)$$

Substitute value of E(s) from eq. 1 to 3

$$C(s) = G(s).(R(s) - B(s))$$

$$\therefore C(s) = G(s).R(s) - G(s).B(s) ------(4)$$

Substitute value of B(s) from eq. 2 to 4

$$C(s) = G(s)R(s) - G(s).H(s).C(s)$$

$$G(s).R(s) = C(s) + G(s).H(s).C(s)$$

$$G(s).R(s) = C(s)(1 + G(s).H(s))$$

Transfer function is given by;

$$\text{T.F.} = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s).H(s)}$$

Laplace Transform of Passive Element (R,L & C)

- The Laplace transform can be used independently on different circuit elements, and then the circuit can be solved entirely in the S Domain (Which is much easier).

- Let's take a look at some of the circuit elements

Laplace Transform of R

- Resistors are time and frequency invariant. Therefore, the transform of a resistor is the same as the resistance of the resistor.

$$L\{Resistor\}=R(s)$$

Laplace Transform of C

Let us look at the relationship between voltage, current, and capacitance, in the time domain:

$$i(t) = C\,\frac{dv(\mathrm{t})}{dt}$$

Solving for voltage, we get the following integral:

$$\mathrm{v(t)} = \frac{1}{C}\int_{to}^{\infty} \mathrm{i(t)\,dt}$$

Then, transforming this equation into the Laplace domain, we get the following:

$$V(s) = \frac{1}{C}\frac{1}{s}I(\mathrm{s})$$

Laplace Transform of C

Again, if we solve for the ratio V(s)/I(s), we get the following:

$$\frac{V(s)}{I(s)} = \frac{1}{sC}$$

Therefore, the transform for a capacitor with capacitance C is given by:

$$L\{\text{capacitor}\} = \frac{1}{sC}$$

Laplace Transform of L

Let us look at the relationship between voltage, current, and inductance, in the time domain:

$$v(\mathrm{t}) = \mathrm{L}\frac{di(\mathrm{t})}{dt}$$

putting this into the Laplace domain, we get the formula:

$$V(\mathrm{s}) = \mathrm{sLI(s)}$$

And solving for our ratio

$$\frac{V(\mathrm{s})}{\mathrm{I(s)}} = \mathrm{sL}$$

Laplace Transform of L

Therefore, the transform of an inductor with inductance L is given by:

$$L\{\text{inductor}\} = \text{sL}$$

Order of System

- The order of control system is defined as the highest power of s present in denominator of closed loop transfer function G(s) of unity feedback system.

Example1: Determine order of given system

$$TF = G(s) = \frac{s(s+2)}{s^4 + 7s^3 + 10s^2 + 5s + 5}$$

Example1: Determine order of given system

$$TF = G(s) = \frac{s(s+2)}{s^4 + 7s^3 + 10s^2 + 5s + 5}$$

Answer: The highest power of equation in denominator

of given transfer function is '4'.

Hence the order of given system is fourth

System Order and Proper System

- Highest power of s present in denominator of closed loop transfer function is called as "Order of System".
- A **proper system** is a system where the degree of the denominator is larger than or equal to the degree of the numerator polynomial.

Example 2 : Determine order of given system

$$G(s) = \frac{(s+5)(s+2)}{s(s+3)(s+4)}$$

Example 2 : Determine order of given system

$$G(s) = \frac{(s+5)(s+2)}{s(s+3)(s+4)}$$

Solution: To obtain highest power of denominator, Simplify denominator polynomial.

$$s(s+3)(s+4) = 0$$

$$s(s^2 + 7s + 12) = 0$$

$$s^3 + 7s^2 + 12s = 0$$

The highest power of equation in denominator of given transfer function is '3'. Hence given system is **"Third Order system"**. The degree of denominator is larger than the numerator hence system is **"Proper System"**

Example 3 : Determine order of given system

$$G(s) = \frac{K(s+5)}{s^3(7s^2 + 12s + 5)}$$

Example 3 : Determine order of given system

$$G(s) = \frac{K(s+5)}{s^3(7s^2+12s+5)}$$

Solution: To obtain highest power of denominator, Simplify denominator polynomial.

$$s^3(7s^2+12s+5)=0$$

$$7s^5+12s^4+5s^3=0$$

The highest power of equation in denominator of given transfer function is '5'. Hence given system is **"Fifth Order system"**. The degree of denominator is larger than the numerator hence system is **"Proper System"**

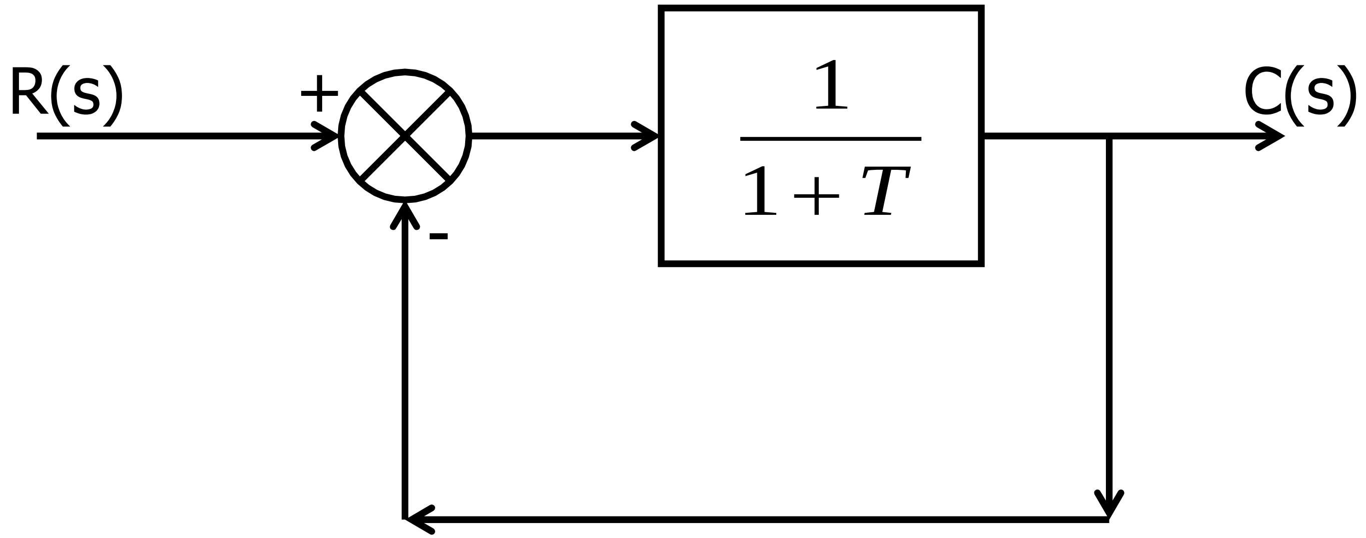
Types of System

(depending on highest power of denominator)

- Zero (0) Order System
- First Order System
- Second Order System

Zero (0) Order System

Definition: If highest power of complex variable 's' present in Characteristics equation is zero, then it is called as "Zero order System"



Zero (0) Order System

Consider a unity feedback system with transfer function

$$G(s) = \frac{1}{1+T}$$

Hence characteristics equation is given by,

$$1+T=0$$

or

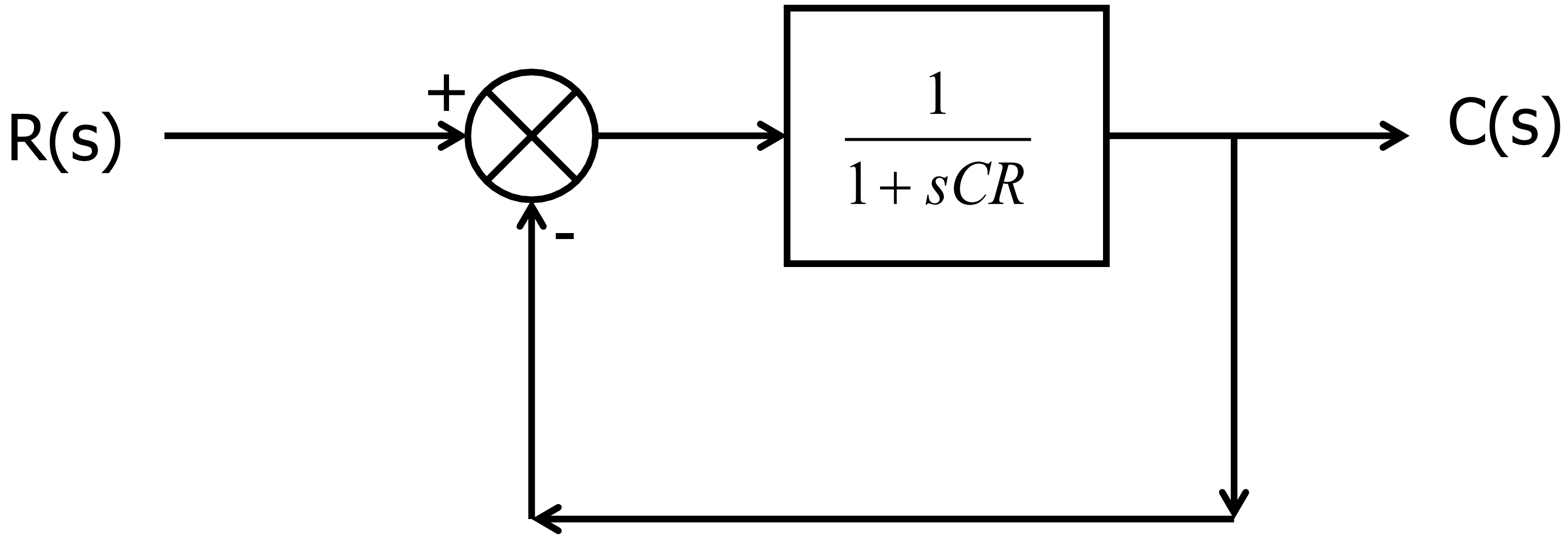
$$1+s^0T=0$$

Here the highest power of s is equal to 0,
Hence the system given above is zero order system.

Practical Example: Amplifier type control system

First Order System

Definition: If highest power of complex variable 's' present In Characteristics equation is one, then it is called as "First order System"



First Order System

Consider a unity feedback system with transfer function

$$G(s) = \frac{1}{1 + sCR}$$

Hence characteristics equation is given by,

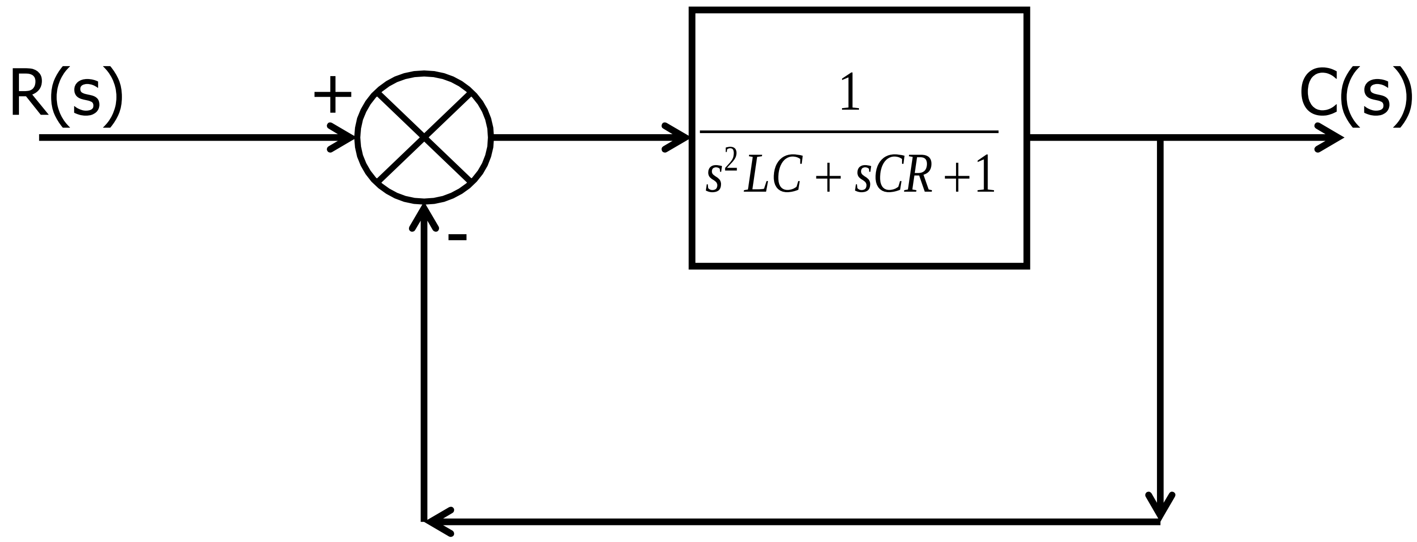
$$1 + sCR = 0$$

Here the highest power of s is equal to 1,
Hence the system given above is First order system.

Practical Example: RC circuits, thermal type systems

Second Order System

Definition: If highest power of complex variable 's' present In Characteristics equation is two, then it is called as "Second order System"



Second Order System

Consider a unity feedback system with transfer function

$$G(s) = \frac{1}{s^2 LC + sCR + 1}$$

Hence characteristics equation is given by,

$$s^2 LC + sCR + 1 = 0$$

Here the highest power of s is equal to 2,
Hence the system given above is Second order system.

Practical Example: RLC circuits, Robotic control system.

Control Systems

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Content

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- Types of Control Systems
  - Open loop control systems
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- Examples of Control Systems
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Introduction

- A **control system** is a control system for a process or plant, wherein control elements are distributed throughout the system. This is in contrast to non-distributed systems, which use a single controller at a central location. In a DCS, a hierarchy of controllers is connected by communications networks for command and monitoring.
- A type of automated control system that is distributed throughout a machine to provide instructions to different parts of the machine. Instead of having a centrally located device controlling all machines, each section of a machine has its own computer that controls the operation. For instance, there may be one machine with a section that controls dry elements of cake frosting and another section controlling the liquid elements, but each section is individually managed by a DCS. A DCS is commonly used in manufacturing equipment and utilizes input and output protocols to control the machine.
- Collection of hardware and instrumentation necessary for implementing control systems.
- Provide the infrastructure (platform) for implementing advanced control algorithms.

DEFINITION

It means by which a variable quantity or set of variable quantities is made to conform to a prescribed norm. It either holds the values of the controlled quantities constant or causes them to vary in a prescribed way.

A control system may be operated by electricity, by mechanical means, by fluid pressure (liquid or gas), or by a combination of means. When a computer is involved in the control circuit, it is usually more convenient to operate all of the control systems electrically, although intermixtures are fairly common.

Basic Components of Control System

- Plant
- Feedback
- Controller
- Error detector

- **Plant:** The portion of a system which is to be controlled or regulated is called as plant or process. It is a unit where actual processing is performed and if we observe in the above figure, the input of the plant is the controlled signal generated by a controller. A plant performs necessary actions on a controlled system and produces the desired output.
- **Feedback:** It is a controlled action in which the output is sampled and a proportional signal is given to the input for automatic correction of any changes in the desired output.

  The output is given as feedback to the input for correction i.e. information about output is given to input for correcting the changes in output due to disturbances. The feedback signal is fed to the error detector. Negative feedback is preferred as it results in better stability and accuracy. The other disturbance signals are rejected.

- **Error Detector:** The function of error detector is to compare the reference input with the feedback signal. It produces an error signal which is a difference of two inputs which are reference signal and a feedback signal. The error signal is fed to the controller for necessary controlled action. This error signal is used to correct the output if there is a deviation from the desired value.
- **Controller:** the element of a system within itself or external to the system which controls the plant is called as a controller. The error signal will be a weak signal and so it has to be amplified and then modified for better control action.

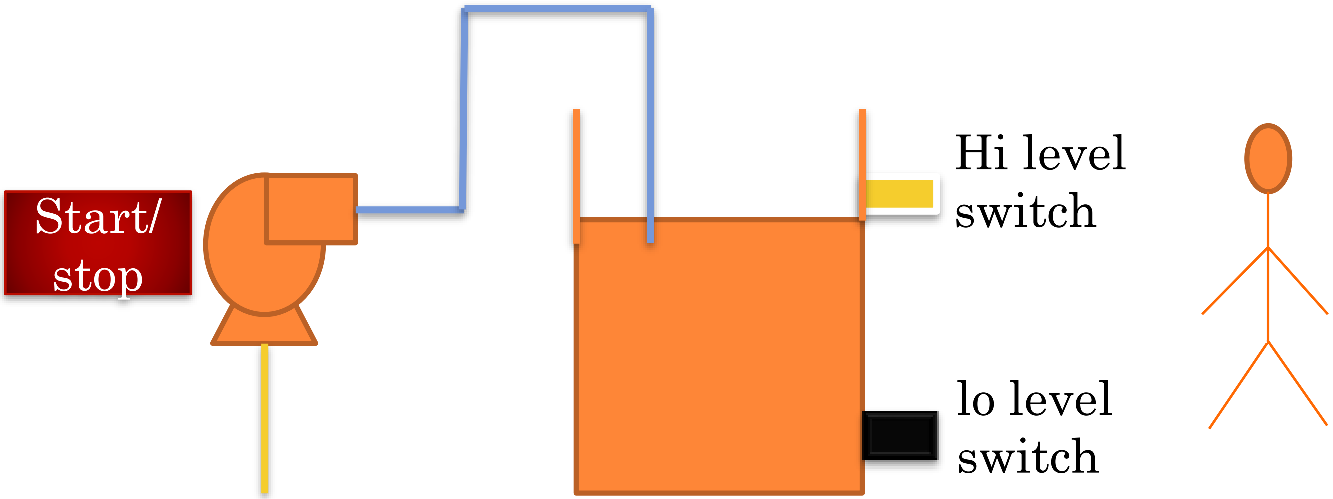
  In most of the systems, the controller itself amplifies the error signal and integrates or differentiates to generate a control signal. An amplifier is used to amplify the error signals and the controller modifies the error signal.

Types of Control Systems

- **Open loop control systems**
- **Closed loop control system**
- Open loop control systems

OPEN LOOP CONTROL SYSTEM

- In open loop control system we have a process which we have to control and some input to change the process and out put. We have an example of a tank level control



Open loop control

- In open loop control system when we start the pump it will continue fill the fluid in the tank but at a time tank will overflow still pump will not stop. In open loop control we have no feedback that what is going on in process.
- We have to manually control the pump by putting a man at near the tank .He will see that if the high level switch glow then he will stop the pump and if lo level will glow then he will start the pump.

Advantages of Open Loop System

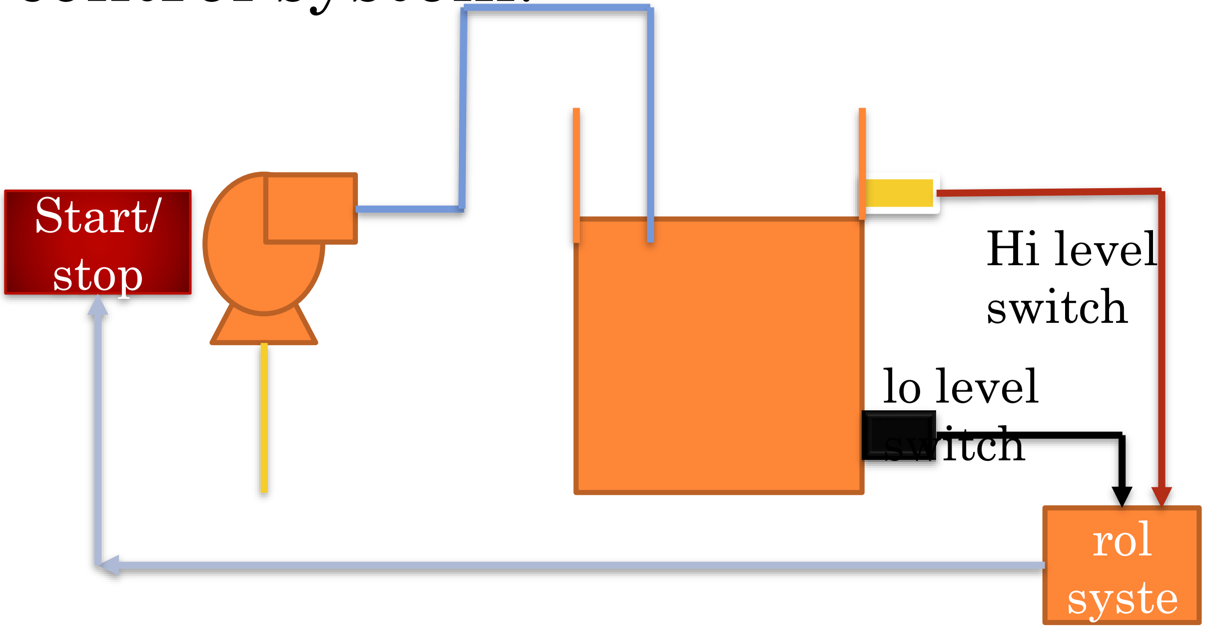
- The open loop systems are simple and economical.
- They are easier to construct.
- The open loop systems are stable.

Disadvantages of Open Loop System

- The open loop systems are inaccurate and unreliable.
- The changes in the output due to external disturbances are not corrected automatically.

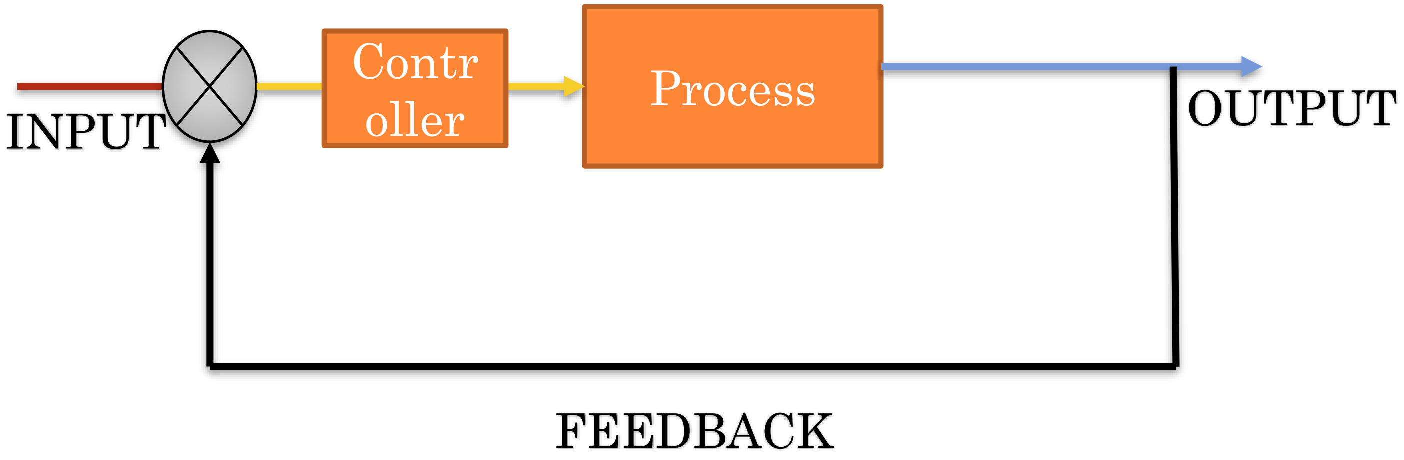
Closed loop control

- Closed loop control system have information about the change in process with respect to change in input. Now consider the previous example in open loop control system.



Close loop control system block diagram

Block diagram



Closed loop control

- In open loop control system when we start the pump we have no status of the tank level but in closed loop control we have status of tank level and if tank level goes below, low level switch act and the pump will start by controller.
- In second case if the tank level goes high then the high level switch act and controller stop the pump. Hence the difference between the open loop and closed loop control system

Advantages of Closed Loop System

- Closed loop systems are accurate.
- They are accurate even in the presence of nonlinearity.
- They are more stable.
- They are less affected by noise.

Disadvantages of Closed Loop System

- They are complex and expensive.
- The feedback in closed loop system may leave to oscillatory response.
- More care is needed to design a closed loop system.
- The overall gain of the system is reduced due to feedback.

Examples of Control Systems

- Distinct examples of control systems are as follows:
- Liquid level control system
- Room temperature control system
- Traffic control system
- Home heating systems

Introduction to control System

By
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BGSIT
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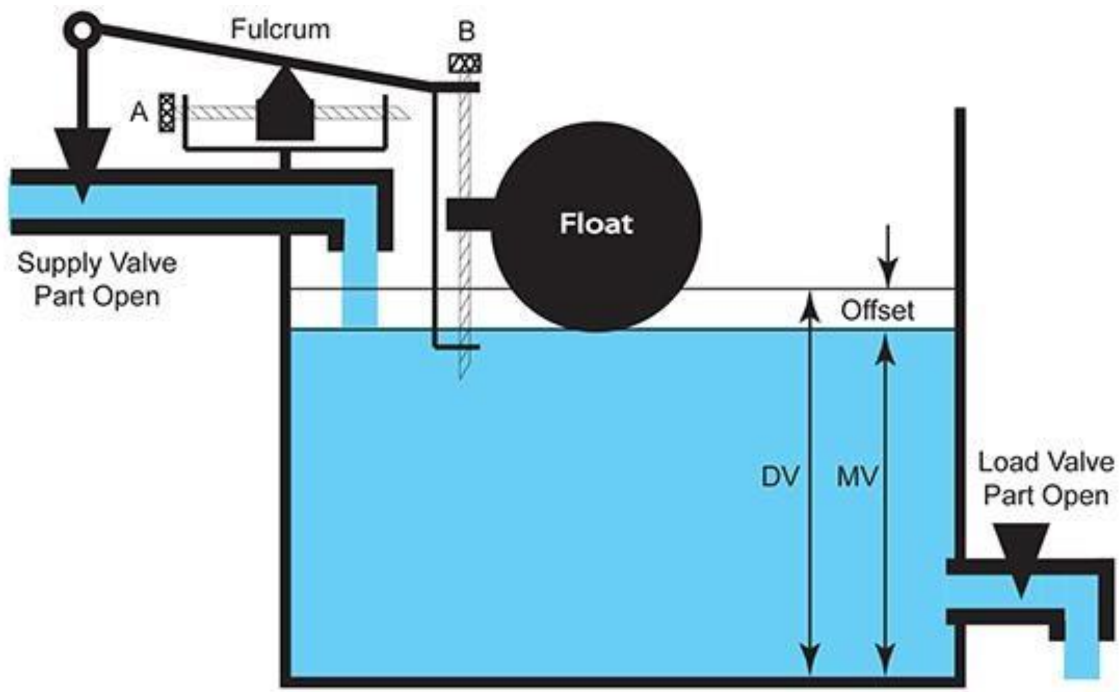
- Concept of automatic controls
- Open loop and closed loop systems
- Concepts of feedback systems
- Requirements of an ideal control system
- Types of controllers
  - Proportional,
  - Integral
  - Proportional Integral,
  - Proportional Integral Differential controllers

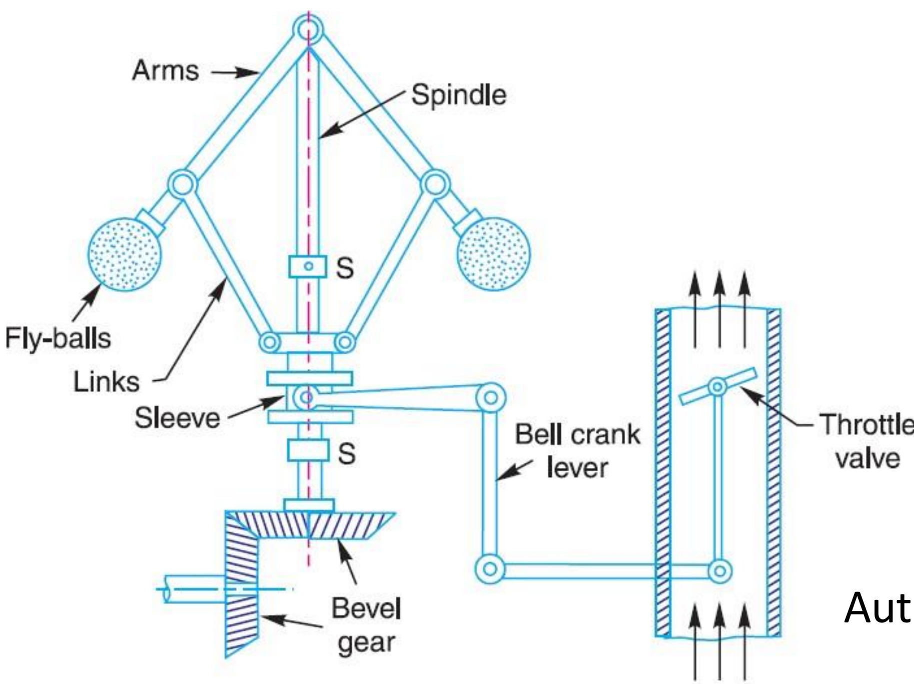
- Automatic control has played a vital role in the advance of engineering and science.
- It is more important in space-vehicle systems, missile-guidance systems, robotic systems, modern manufacturing and industrial processes.
- For example,
  - Numerical control of machine tools in the manufacturing industries.
  - Design of autopilot systems in the aerospace industries
  - Design of cars and trucks in the automobile industries.
  - Speed Governors

- It is also essential in industrial operations as
  - controlling pressure,
  - temperature,
  - humidity,
  - viscosity, and
  - flow in the process industries.
- Automatic control helps in attaining optimal performance of dynamic systems, improving productivity, relieving the drudgery of many routine repetitive manual operations.

Concept of automatic controls
Examples of Automatic control systems



Automatic water lever controller



Automatic Engine speed controller

Concept of automatic controls

- Automatic control uses application of mechanisms to the operation and regulation of processes without continuous direct human intervention.
- This assumes the process remains in the normal range for the control system.
- An automatic control system has two process variables associated with it:
  - a controlled variable
  - a manipulated variable.
- A controlled variable is the process variable that is maintained at a specified value or within a specified range.
- In the previous example, the storage tank level is the controlled variable.

Concept of automatic controls

- A manipulated variable is the process variable that is acted on by the control system to maintain the controlled variable at the specified value or within the specified range.
- The flow rate of the water supplied to the tank is the manipulated variable.

Functions of Automatic Control

- In any automatic control system, the four basic functions that occur are:
  - Measurement
  - Comparison
  - Computation
  - Correction

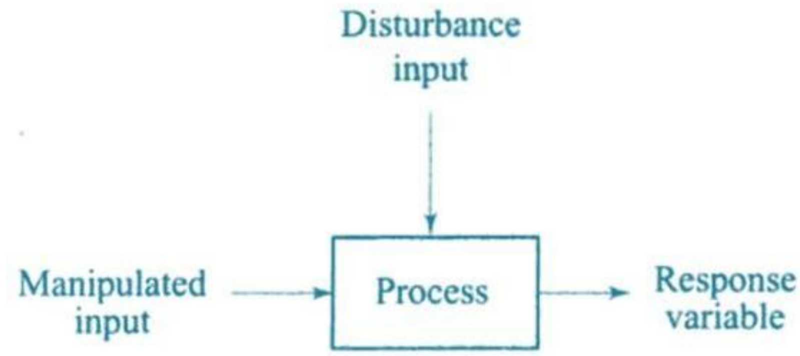
Basic Definitions

System :

A system is a combination or an arrangement of different physical components which act together as a entire unit to achieve certain objective.

- E.g.,
  - A classroom is a physical system. A room along with the combination of benches, blackboard, fans, lighting arrangement etc. can be called as a classroom which acts as elementary system.
  - In a classroom, professor is delivering his lecture, it becomes a control system as; he tries to regulate, direct or command the students in order to achieve the objective which is to input good knowledge to the students.

Basic Definitions



Plant :

- The portion of a system which is to be controlled or regulated is called as the plant.
- A plant may be a piece of equipment, perhaps just a set of machine parts.
- The purpose of plant is to perform a particular operation.
- E.g., mechanical device, a heating furnace, a chemical reactor, or a spacecraft.

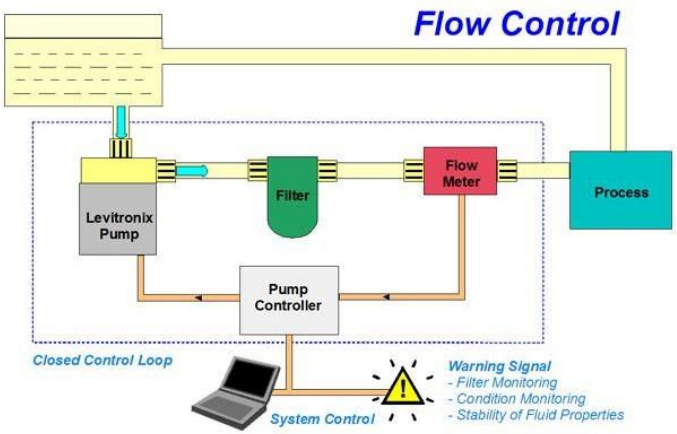
Process:

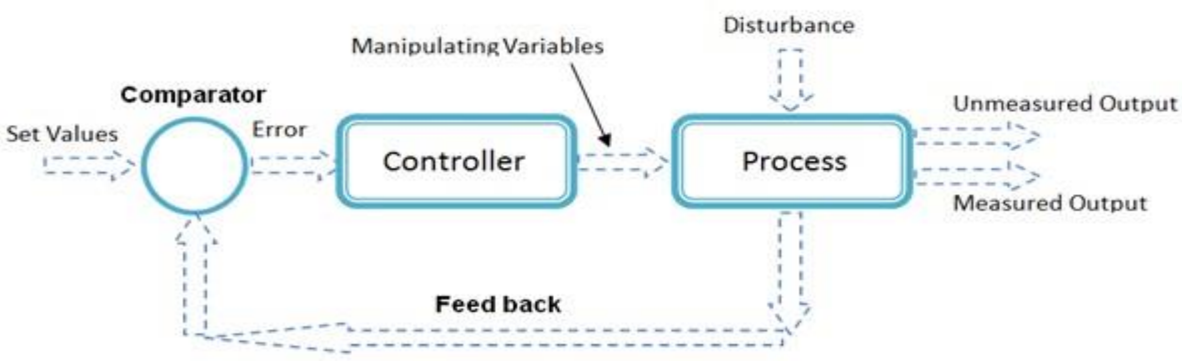
- Any operation to be controlled is called a process.
- Examples are chemical, economic, and biological processes.

Basic Definitions

Controller :

- The element of the system itself or external to the system which controls the plant or the process is called as controller.
- E.g., ON/OFF switch to control bulb.





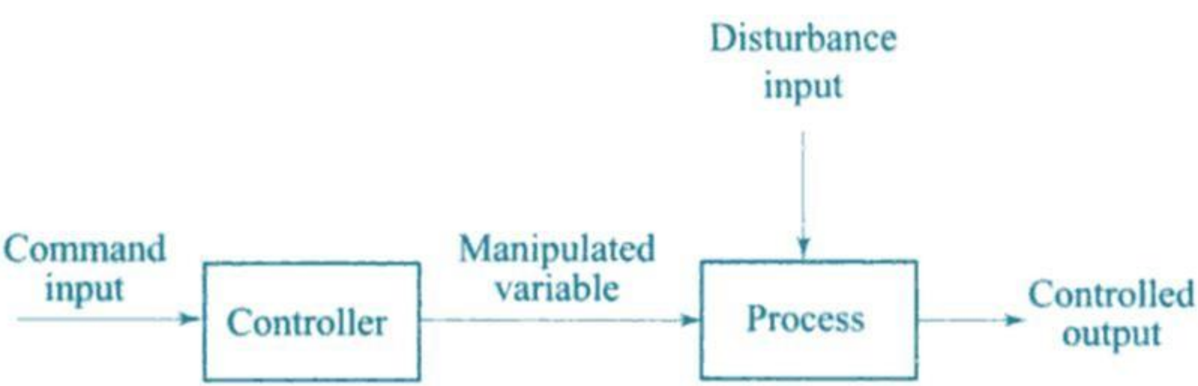
Basic Definitions

Input :

- It is an applied signal or an excitation signal applied to control system from an external energy source in order to produce a specified output.
- For each system, there must be excitation and system accepts it as an input

Output :

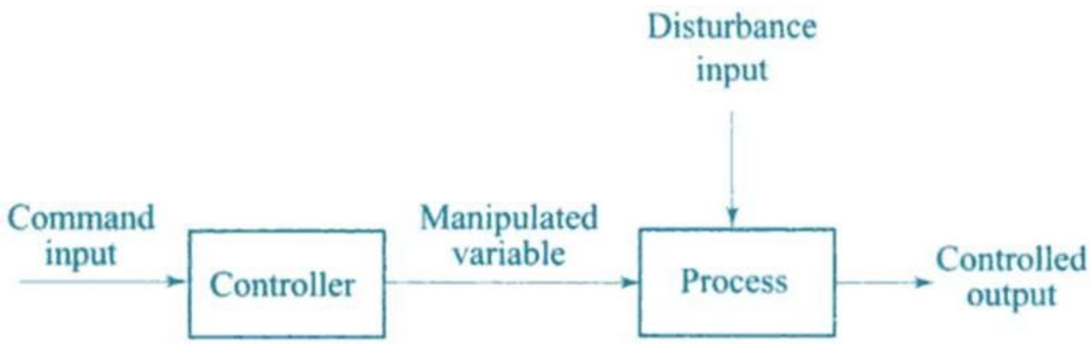
- It is the particular signal of interest or the actual response obtained from a control system when input is applied to it.
- for analysing the behaviour of system for such input, it is necessary to define the output of a system.



Basic Definitions

Disturbances :

- Disturbance is a signal which tends to adversely affect the value of the output of a system.
- Disturbances are undesirable and unavoidable effects beyond our control, generated from outside process-environment, and from within.
- If such a disturbance is generated within the system itself, it is called as internal disturbance.
- The disturbance generated outside the system acting as an extra input to the system in addition to its normal input, affecting the output adversely is called as an external disturbance.
- The presence of the disturbance is one of the main reasons of using control.



Classification of Control Systems

1) Natural Control System
— Universe
— Human Body

2) Manmade Control System
— Vehicles
— Aeroplanes

3) Manual Control Systems
   - Room Temperature regulation Via Electric Fan
   - Water Level Control

4) Automatic Control System
   - Room Temperature regulation Via A.C
   - Human Body Temperature Control

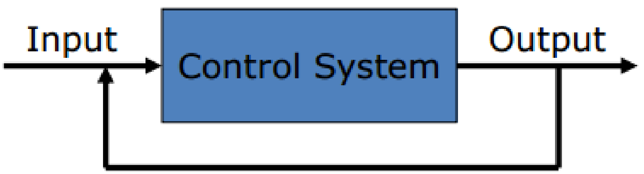
Classification of Control Systems

5) Open-Loop Control System

- Washing Machine
- Toaster
- Electric Fan

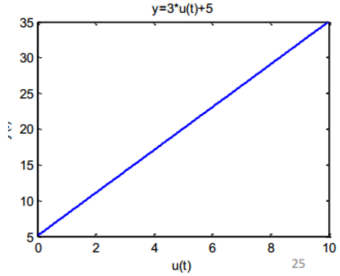
6) Closed-loop Control System

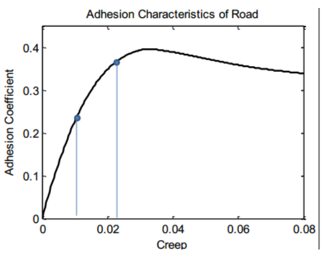
- Refrigerator
- Auto-pilot system
- Driverless cars



7) Linear Vs Nonlinear Control System

A Control System in which output varies linearly with the input is called a linear control system.





Classification of Control Systems

8) Time invariant vs Time variant

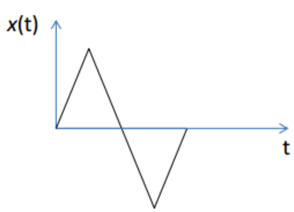
- When the characteristics of the system do not depend upon time itself then the system is said to time invariant control system.
- Time varying control system is a system in which one or more parameters vary with time.

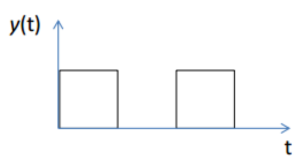
9) Continuous Data Vs Discrete Data System

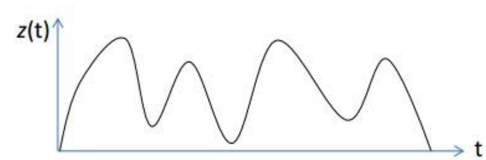
- In continuous data control system all system variables are function of a continuous time t.
- A discrete time control system involves one or more variables that are known only at discrete time intervals.

10) Deterministic vs Stochastic Control System

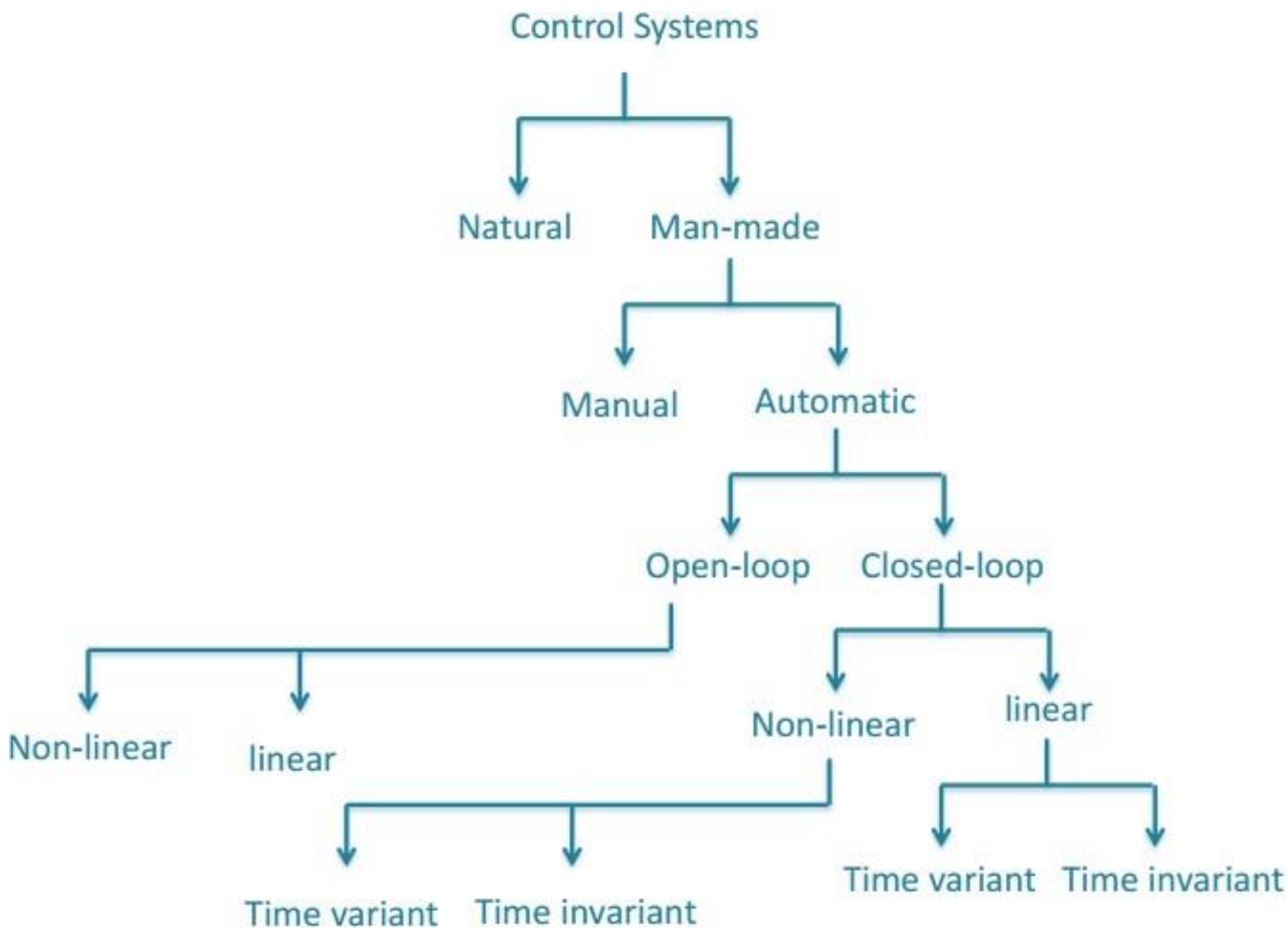
- A control System is deterministic if the response to input is predictable and repeatable.
- If not, the control system is a stochastic control system.





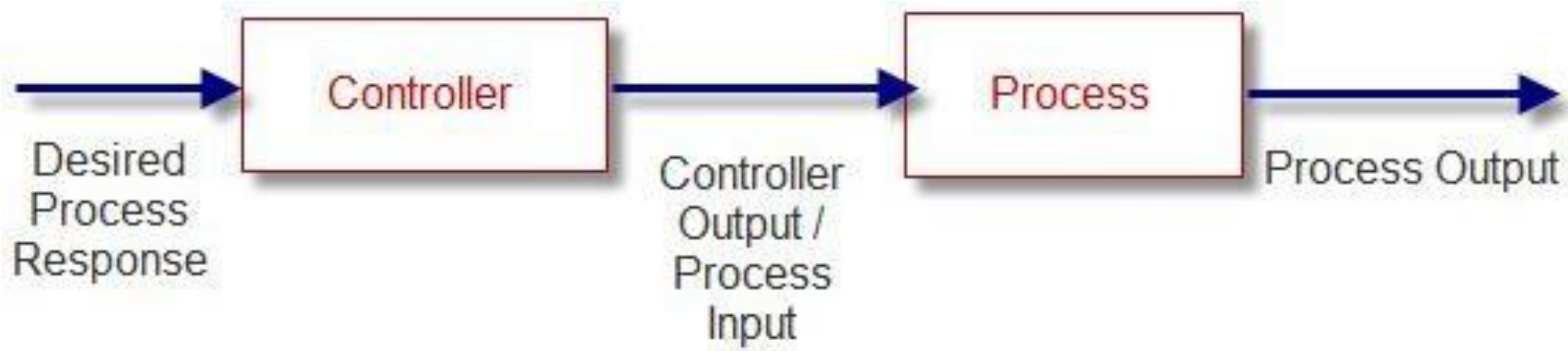


Classification of Control Systems



Open loop control systems

- Any physical system which does not automatically correct for variation in its output, is called an open-loop system.
- Such a system may be represented by the block diagram as shown in Fig.



- In these systems, output is dependent on input but controlling action or input is totally independent of the output or changes in output of the system.
- In these systems the output remains constant for a constant input signal **provided the external conditions remain unaltered.**

Open loop control systems

- In any open-loop control system the output is not compared with the reference input. As a result, the accuracy of the system depends on calibration.
- In the presence of disturbances, an open-loop control system will not perform the desired task. Open-loop control can be used, in practice, only if the relationship between the input and output is known and if there are neither internal nor external disturbances.
- Clearly, such systems are not feedback control systems. Note that any control system that operates on a time basis is open loop.
- For instance, traffic control by means of signals operated on a time basis is an example of open-loop control.

Open loop control systems

Advantages: The advantages of open loop control system are,

1) Such systems are simple in construction.

2) Very much convenient when output is difficult to measure.

3) Such systems are easy from maintenance point of view.

4) Generally these are not troubled with the problems of stability.

5) Such systems are simple to design and hence economical.

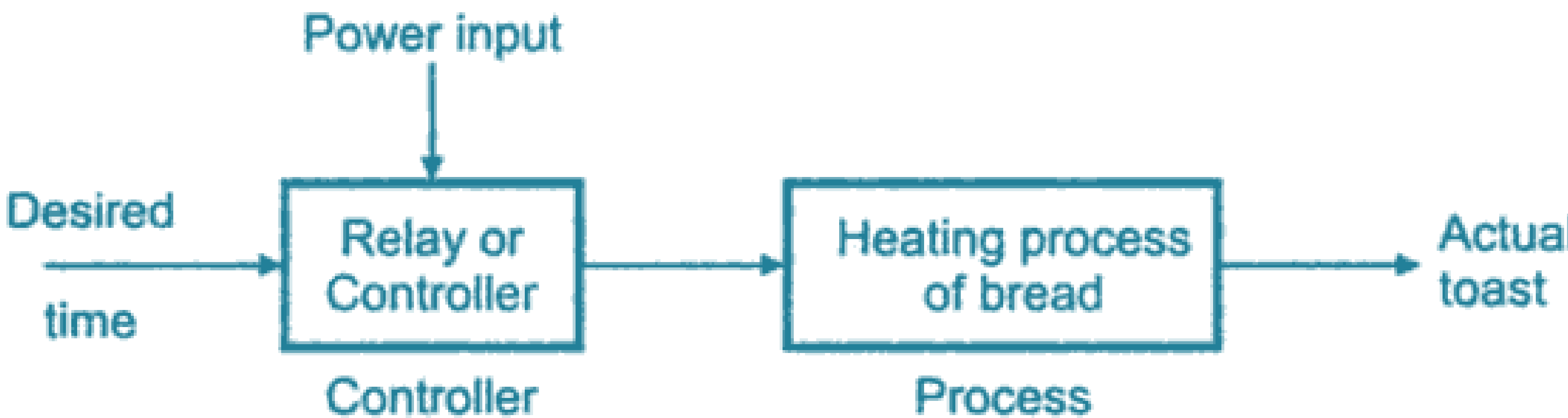
Disadvantages: The disadvantages of open loop control system are,

1. These systems are inaccurate and unreliable because accuracy of such systems are totally dependent on the accurate pre-calibration of the controller.
2. These systems give inaccurate results if there are variations in the external environment.
3. These systems cant sense internal disturbances in the system, after the controller stage.
4. Recalibration of the controller is necessary, time to time to maintain the quality and accuracy of the desired output.

Examples of an open loop system

1) Automatic Toaster System

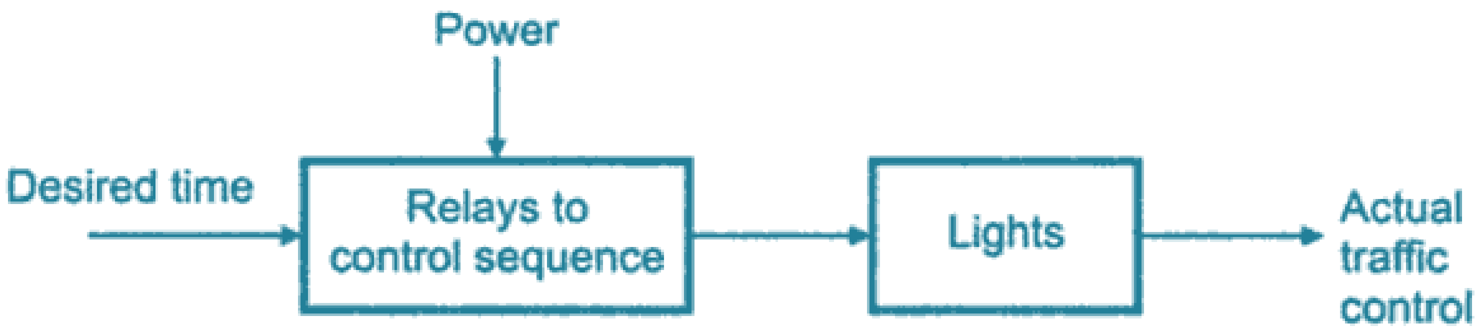
- In this system, the quality of toast depends upon the time for which the toast is heated.
- Depending on the time setting, bread is simply heated in this system.
- The toast quality is to be judged by the user and has no effect on the inputs.



Examples of an open loop system

2) Traffic Light Controller

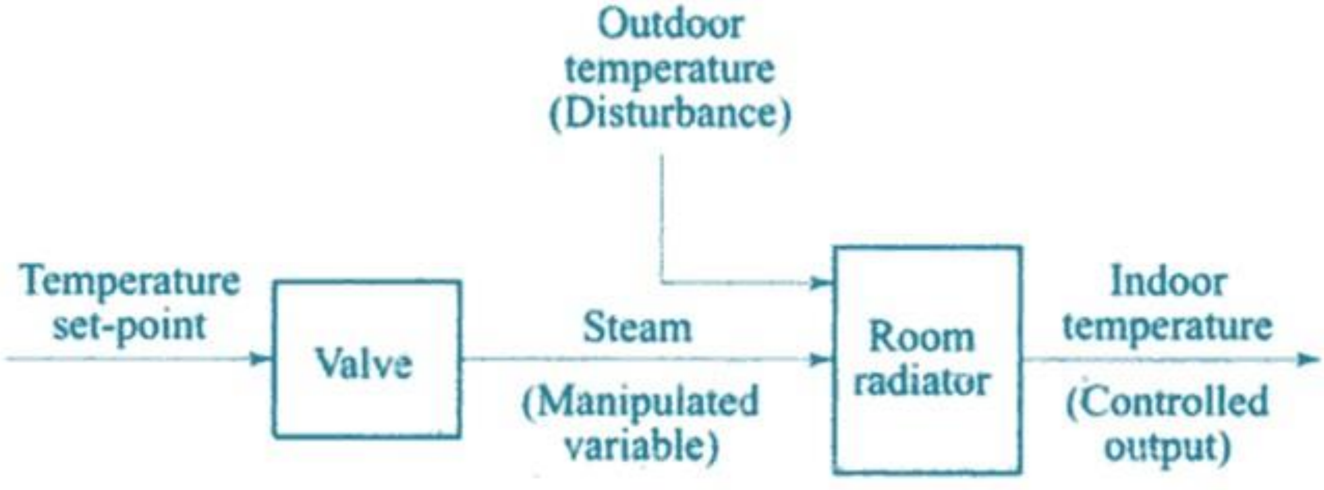
- A traffic flow control system used on roads is time dependent.
- The traffic on the road becomes mobile or stationary depending on the duration and sequence of lamp glow.
- The sequence and duration are controlled by relays which are predetermined and not dependent on the rush on the road.



Closed loop control systems

3) Residential Heating System

- The indoor temperature is the response variable of interest, and it is affected by the main disturbance input—the outdoor temperature.
- The desired temperature is set on a calibrated dial. This positions the valve that admits the steam for circulation through the radiator.
- The valve dial is calibrated when the environment temperature has certain value.
- When this value changes significantly, the controlled temperature will deviate from the desired value by a large error and hence precise control will not be realized.



Closed loop control systems

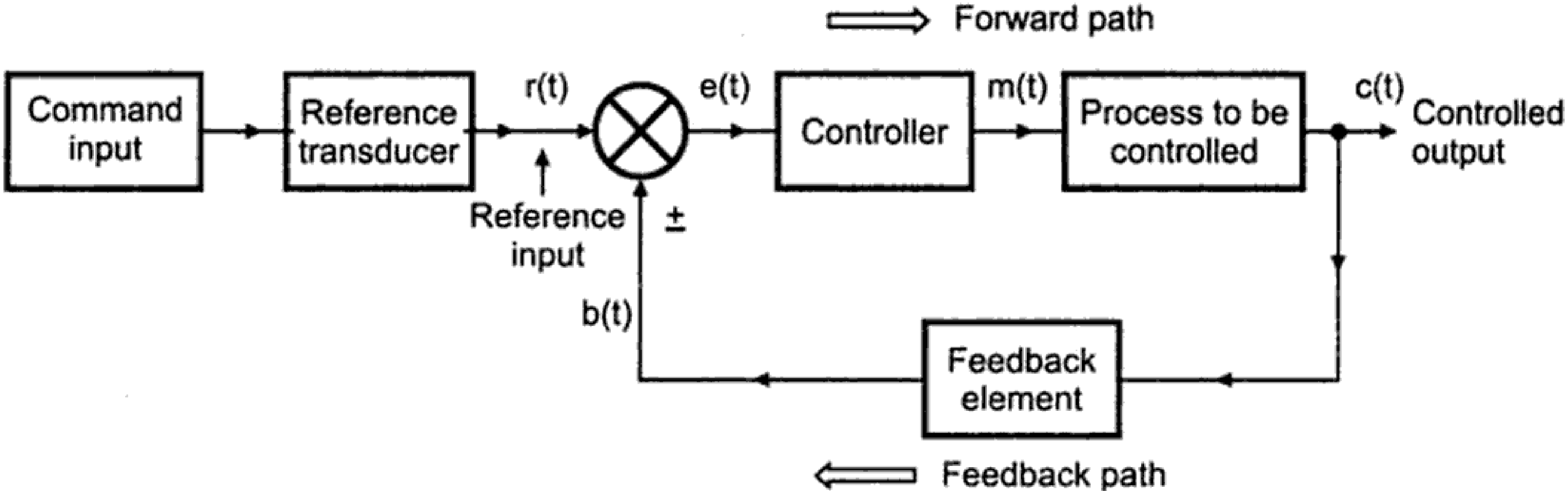
Feedback Control Systems.

- A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a feedback control system.

Closed-Loop Control Systems.

- Feedback control systems are often referred to as closed-loop control systems.
- In practice, the terms feedback control and closed-loop control are used interchangeably.
- In a closed-loop control system the actuating error signal (which is the difference between the input signal and the feedback signal) is fed to the controller so as to reduce the error and bring the output of the system to a desired value.
- The term closed-loop control always implies the use of feedback control action in order to reduce system error.

Closed loop control systems



The various signals are,

r(t) = Reference input

e(t) = Error signal

c(t) = Controlled output

m(t) = Manipulated signal

b(t) = Feedback signal

Closed loop control systems

- The part of output, which is to be decided by feedback element is fed back to the reference input. The signal which is output of feedback element is called feedback signal, b(t).
- It is then compared with the reference input giving error signal $e(t) = r(t) \pm b(t)$
- When feedback sign is positive, systems are called positive feedback systems and if it is negative systems are called negative feedback systems.
- This error signal is then modified by controller and decides the proportional manipulated signal for the process to be controlled.
- This manipulation is such that error will approach zero. This signal then actuates the actual system and produces an output. As output is controlled one, hence called controlled output c(t).

Closed loop control systems

Advantages

1. **Accuracy of these systems** is always very high because controller modifies and manipulates the actuating signal such that error in the system will be zero.
2. closed loop system senses environmental changes, as well as internal disturbances and accordingly modifies the error.
3. There is reduced effect of nonlinearities and distortions.
4. Bandwidth (operating frequency zone) for such system is veryhigh.

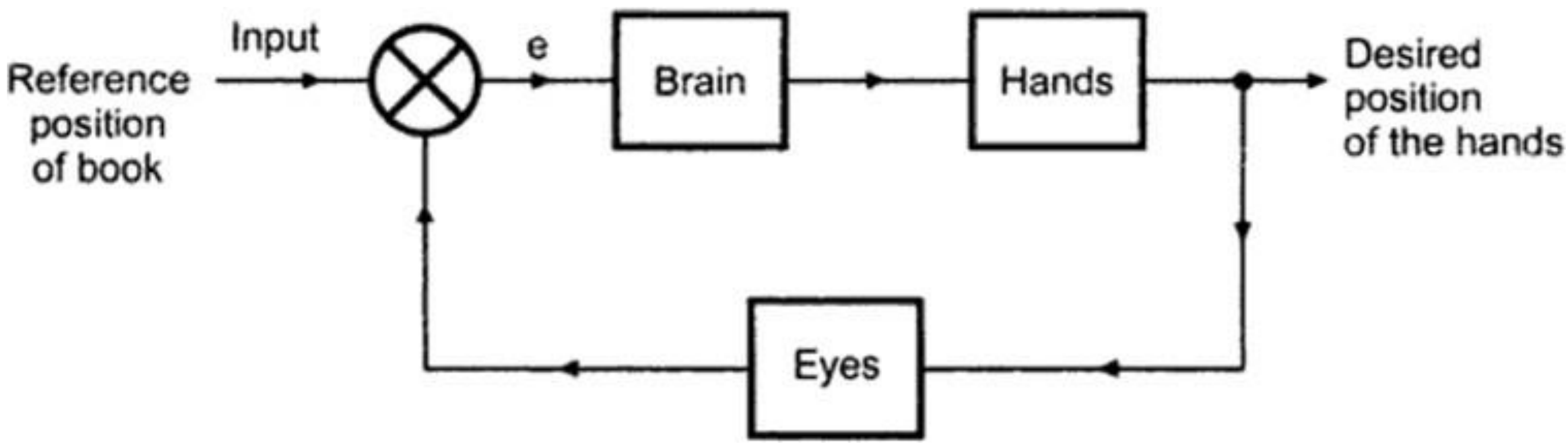
Disadvantages

1. systems are complicated and time consuming from design point of view and hence costlier.
2. Due to feedback, system tries to correct the error from time to time. Tendency to overcorrect the error may cause oscillations without bound in the system.
3. System has to be designed taking into consideration problems of instability due to feedback.
4. The stability problems are severe and must be taken care of while designing the system.

Examples of an closed loop system

1. Human Being

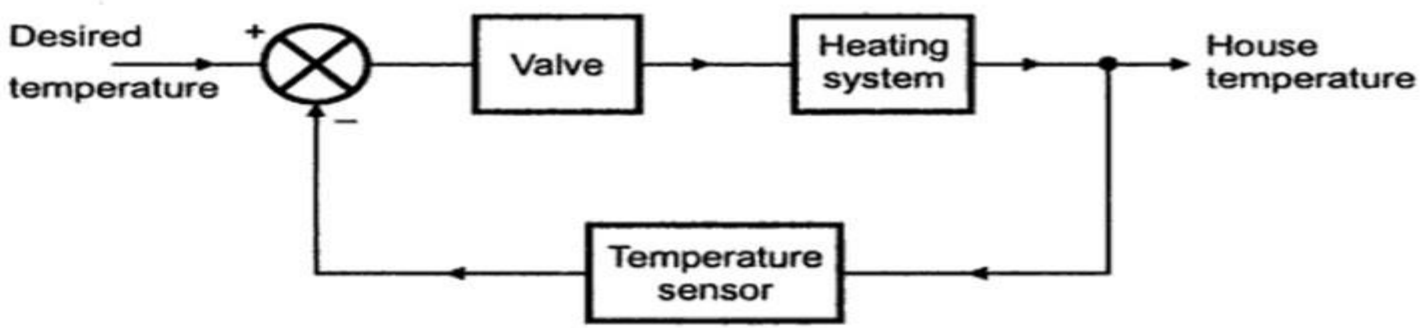
- The best example is human being. If a person wants to reach for a book on the table, Position of the book is given as the reference.
- Feedback signal from eyes, compares the actual position of hands with reference position. Error signal is given to brain.
- Brain manipulates this error and gives signal to the hands. This process continues till the position of the hands get achieved appropriately.



Examples of an closed loop system

2. Home Heating System

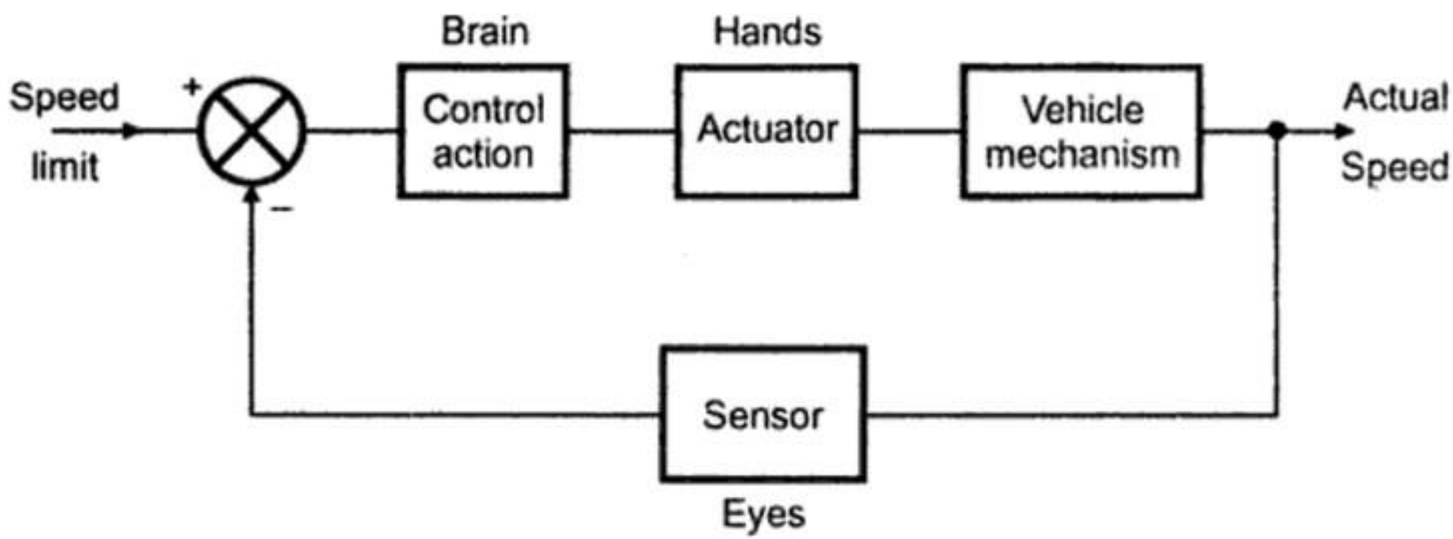
- In this system, the heating system is operated by a valve.
- The actual temperature is sensed by a thermal sensor and compared with the desired temperature.
- The difference between the two, actuates the valve mechanism to change the temperature as per the requirement.



Examples of an closed loop system

3. Manual Speed Control System

- A locomotive operator driving a train is a good example of a manual speed control system.
- The objective is to maintain the speed equal to the speed limits set.
- The entire system is shown in the block diagram in the Fig.



Open Loop	Closed Loop
Any change in output has no effect on the input i.e. feedback does not exists.	Changes in output, affects the input which is possible by use of feedback.
Output measurement is not required for operation of system.	Output measurement is necessary.
Feedback element is absent.	Feedback element is present.
Error detector is absent.	Error detector is necessary.
It is inaccurate and unreliable.	Highly accurate and reliable.
Highly sensitive to the disturbances.	Less sensitive to the disturbances.
Highly sensitive to the environmental changes.	Less sensitive to the environmental changes.
Bandwidth is small.	Bandwidth is large.
Simple to construct and cheap.	Complicated to design and hence costly.
Generally are stable in nature.	Stability is the major consideration while designing
Highly affected by nonlinearities.	Reduced effect of nonlinearities.

Requirements of an Ideal Control Systems

To achieve the required objective, a good control system must satisfy the following requirements.

1. Accuracy :
   - A good control system must be highly accurate.
   - The open loop systems are generally less accurate and hence feedback is introduced to reduce the error in the system.
2. Sensitivity :
   - A good control system should be very insensitive to environmental changes, age etc. But, must be sensitive to the input commands.
   - The performance should not be affected by small changes in the certain parameters of the system.

Requirements of an Ideal Control Systems

3) External disturbance or noise :

- All the physical systems are subjected to external disturbances and noise signals during operation.
- A requirement of a good control system is that system is insensitive to noise and external disturbances but sensitive to the input commands.
- It should be able to reduce the effects of undesirable disturbances.

4) Stability :

- A concept of stability means output of system must follow reference input and must produced bounded output for bounded input.
- A good control system is one which is stable in nature.

Requirements of an Ideal Control Systems

5) Bandwidth :
   - This requirement is related to the frequency response of the system.
   - For the input frequency range, it should give satisfactory output.

6) Speed :
   - A system should have good speed. This means output of the system should approach to its desired value as quickly as possible.
   - System should settled down to its final, value as quickly as possible.

7) Oscillations :
   - The system should exhibits suitable damping i.e. the controlled output should follow the changes in the reference input without unduly large oscillations or overshoots.

Introduction to Controllers

- The concept of a control system is to sense deviation of the output from the desired value and correct it, till the desired output is achieved.
- The deviation of the actual output from its desired value is called an error. The measurement of error is possible because of feedback.
- The feedback allows us to compare the actual output with its desired value to generate the error.
- The controller is an element which accepts the error in some form and decides the proper corrective action.
- The output of the controller is then applied to the process or final control element. This brings the output back to its desired set point value.
- The controller is the heart of a control system. The accuracy of the entire system depends on how sensitive is the controller to the error detected and how it is manipulating such an error.

Classifications of Industrial Controllers

Most industrial controllers may be classified according to their control actions as:

1. Two-position or on-off controllers
2. **Proportional controllers**
3. **Integral controllers**
4. **Proportional-plus-integral controllers**
5. Proportional-plus-derivative controllers
6. **Proportional-plus-integral-plus-derivative controllers**

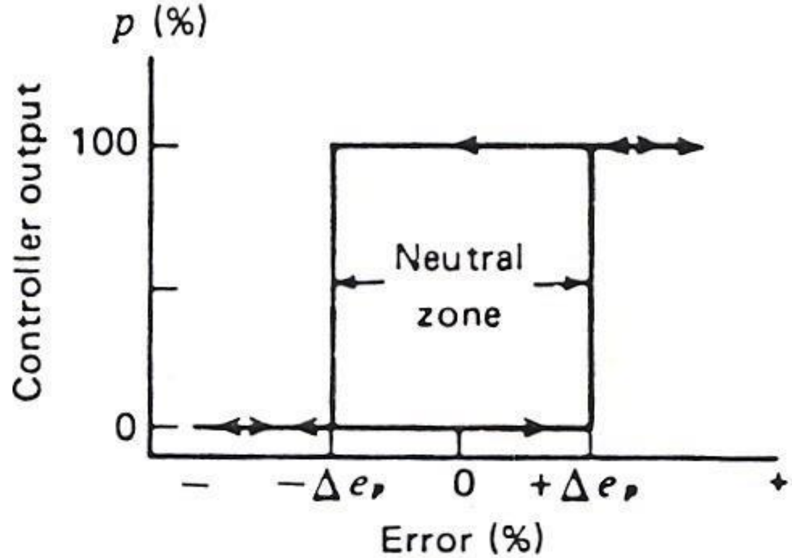
Two-position or on-off controllers

- The most elementary controller mode is the two-position or ON/OFF controller mode.
- It is the simplest, cheapest.
- The most general form can be given by:

$$P = 0\,\% \qquad ep < 0$$

$$P = 100\,\% \qquad ep > 0$$

Ex. ON/OFF switch



- The relation shows that when the measured value is less than the set-point (i.e. ep > 0), the controller output will be full (i.e. 100%),
- when the measured value is more than the setpoint (i.e. ep < 0), the controller output will be zero (i.e. 0%).

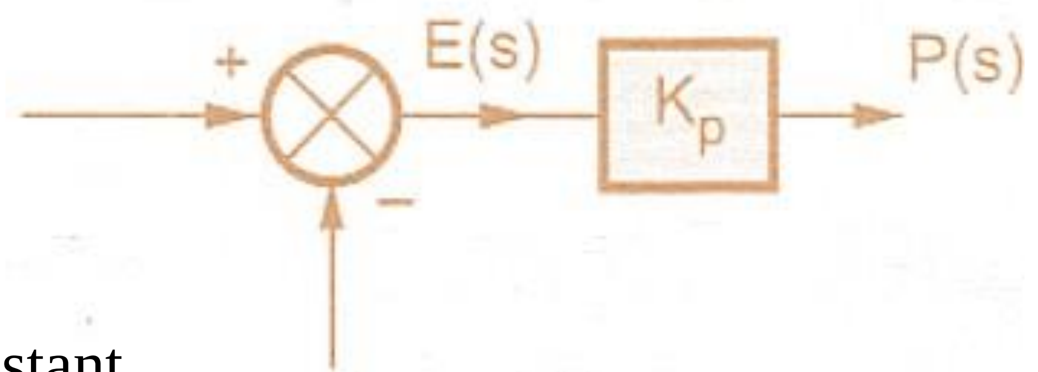
Proportional controllers

- In this control mode, the output of the controller is simple proportional to the error e(t).
- The relation between the error e(t) and the controller output p is determined by constant called proportional gain constant denoted as K_p.
- The output of the controller is a linear function of the error e(t).
- Thus each value of the error has a unique value of the controller output.
- The range of the error which covers 0 % to 100 % controller output is called proportional band.
- The basic relationship between output of the controller and error signal is given by,

$$p(t) = K_p e(t)$$

Taking Laplace transform,

$$P(s) = K_p E(s)$$



K_p = Proportional gain constant

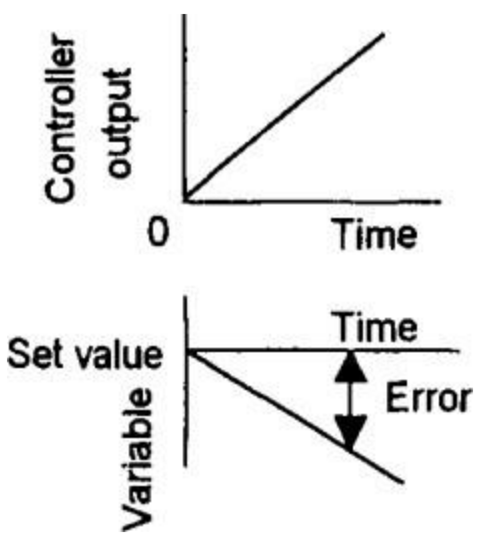
Proportional controllers

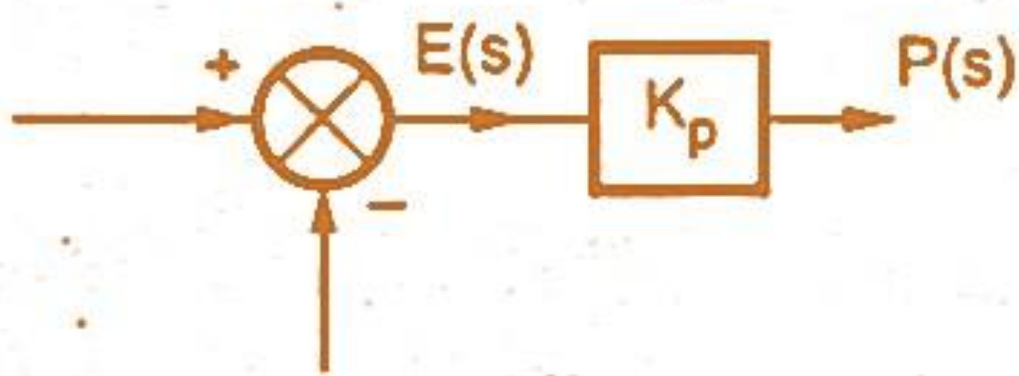
- Though there exists linear relation between controller output and the error, for a zero error the controller output should not be zero, otherwise the process will come to halt.
- Hence there exists some controller output P_o for the zero error. Hence mathematically the proportional control mode is expressed as,

$$p(t) = K_p\, e(t) + P_o$$

K_p = Proportional gain constant

P_o = Controller output with zero error

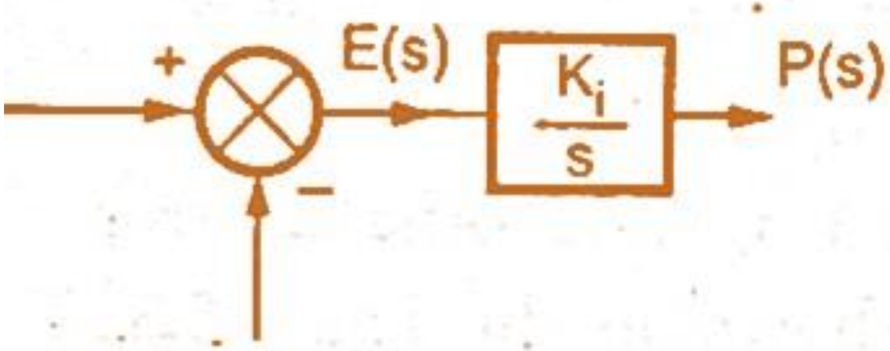




Integral controllers

- In the proportional control mode, error reduces but can not go to zero.
- It finally produces an offset error. It can not adapt with the changing load conditions. To avoid this, another control mode is often used in the control systems which is based on the history of the errors. This mode is called integral mode or reset action controller.
- In such a controller, the value of the controller output p(t) is changed at a rate which is proportioned to the actuating error signal e(t). Mathematically it is expressed as,

$$\frac{dp(t)}{dt} = K_i e(t)$$



K_i = Constant relating error and rate

Taking Laplace transform,

$sP(s) = Ki\ E(s)$ or $P(s) = (Ki/s)\ E(s)$

Integral controllers

- The constant K_i is also called integral constant.
- Integrating the above equation, actual controller output at any time *t* can be obtained as,

$$p(t) = K_i \int_0^t e(t)\,dt + p(0)$$

Where

p(0) = Controller output when integral action starts i.e. at t = 0.

Derivative controllers

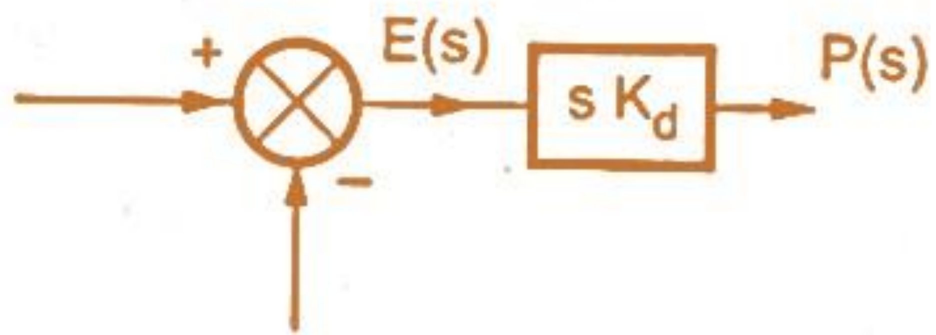
- The controller produces a control action that is proportional to the rate at which the error is changing de(t)/dt.
- The mathematical equation for the mode is,

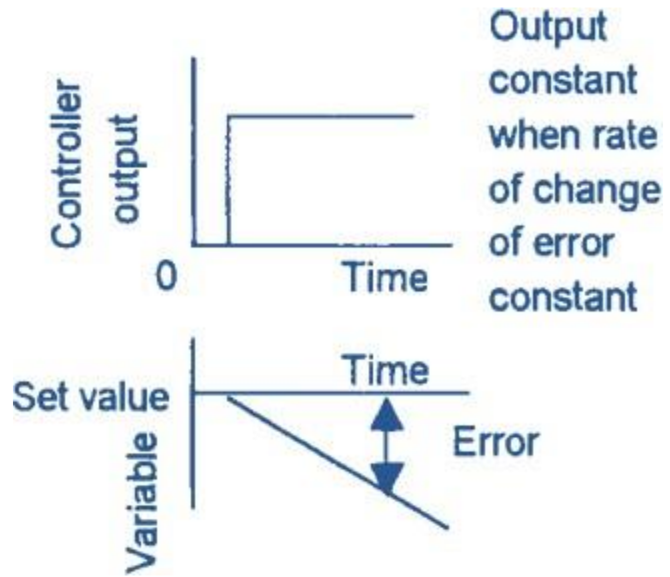
$$p(t) = K_d \frac{de(t)}{dt}$$

where K_d = Derivative gain constant.

Taking Laplace transform

$P(s) = K_d \, s \, E(s)$





Proportional + Integral controllers

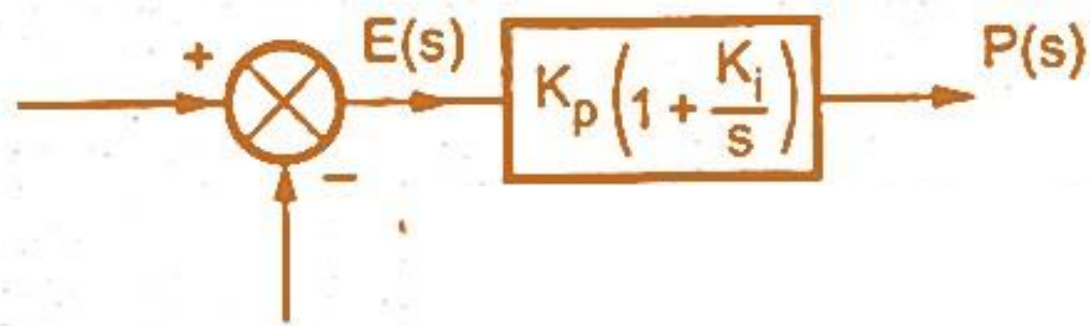
- This is a composite control mode obtained by combining the proportional mode and the integral mode.
- The mathematical expression for such a composite control is,

$$p(t) = K_p e(t) + K_p K_i \int_0^t e(t)\,dt + p(0)$$

Taking Laplace transform,

$$P(s) = \left[K_p + \frac{K_p K_i}{s} \right] E(s)$$

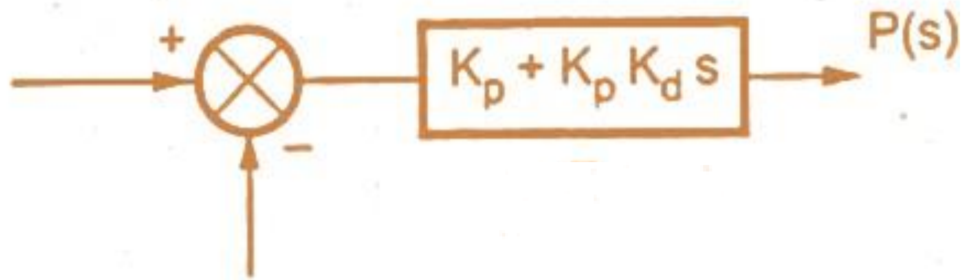
$$P(s) = K_p \left[1 + \frac{K_i}{s} \right] E(s)$$



Proportional + Derivative controllers

- The series combination of proportional and derivative control modes gives proportional plus derivative control mode.
- The mathematical expression for the PD composite control is,

$$p(t) = K_p e(t) + K_p K_d \frac{de(t)}{dt} + p(0)$$



- Taking Laplace transform,

$$P(s) = \left[K_p + K_p K_d s\right] E(s)$$

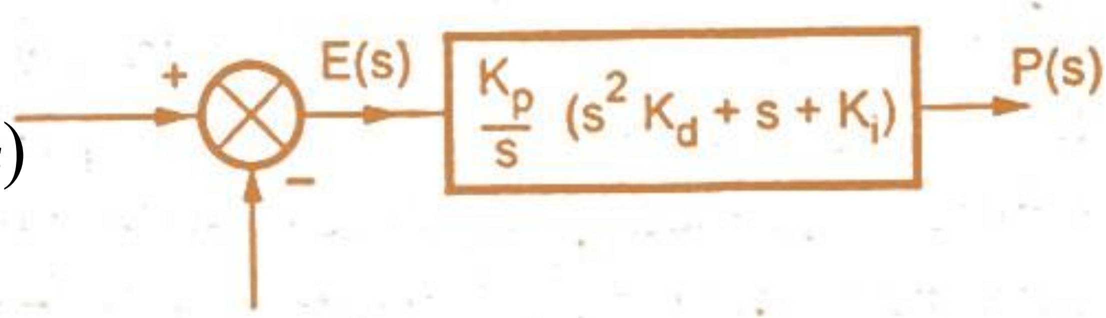
- The addition of a derivative mode to a proportional controller modifies its response to inputs.
- A PD controller provides an element to the response which is largest when the rate of change of the error is greatest and diminishes as it becomes smaller.
- The derivative mode is never used alone because it is not capable of maintaining a control signal under steady error conditions.
- It is always used with the proportional mode and often additionally

Proportional + Integral + Derivative controllers

- The composite controller including the combination of the proportional, integral and derivative control mode is called PID control mode and the controller is called three mode controller.
- It is very much complex to design but very powerful in action.
- Mathematically such a control mode can be expressed as,

$$p(t) = K_p e(t) + K_p K_i \int_0^t e(t)\,dt + K_p K_d \frac{de(t)}{dt} + p(0)$$

$$P(s) = \left[K_p + \frac{K_p K_i}{s} + K_p K_d s \right] E(s)$$

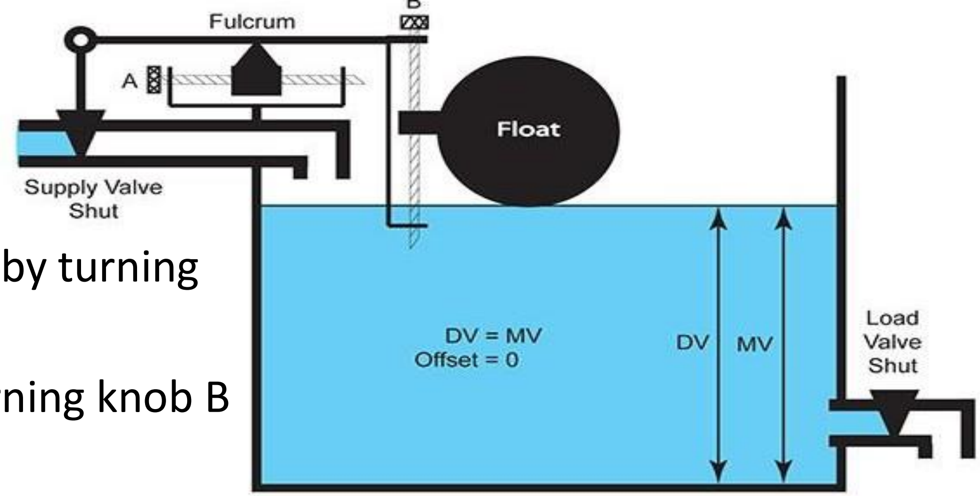


$$P(s) = \frac{K_p}{s}\left[s^2 K_d + s + K_i \right] E(s)$$

Proportional + Integral + Derivative controllers

- This mode has advantages of all the modes.
- The integral mode eliminates the offset error of the proportional mode and the response is also very fast due to derivative mode.
- The sudden response is produced due to derivative mode.
- Thus it can be used for any process condition.
- With the PID control action, there is no offset, no oscillations with least settling time.
- So there is improvement in both transient as well as steady state response.

Example: Water level controller



A The Fulcrum can be adjusted horizontally by turning knob A

B The Float can be adjusted vertically by turning knob B

Desired Value (DV)	=	The required level of water in the tank
Measured Value (MV)	=	The actual level of water in the tank.
Offset or Error (E)	=	The difference between the required and actual level (DV-MV)
Gain (K)	=	The ratio of float movement to valve movement

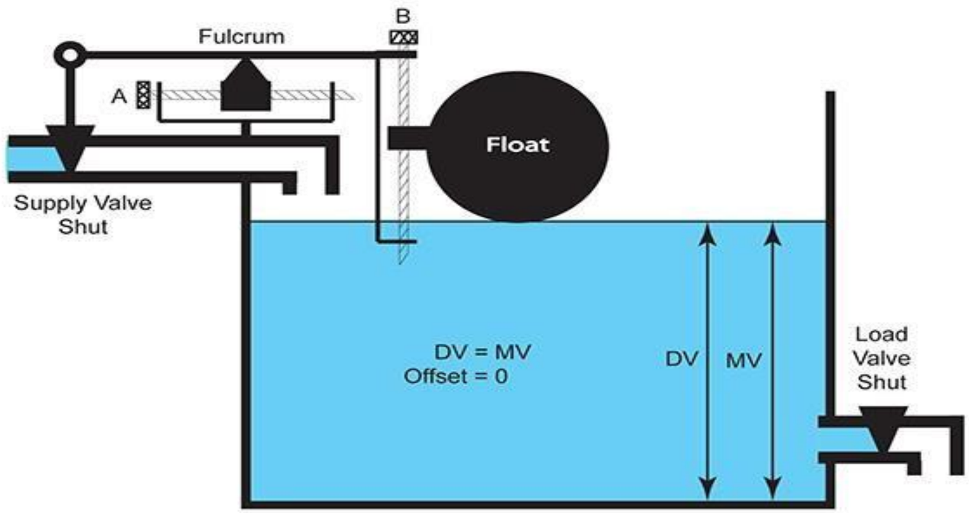
- The water level and the Float continue to drop and the Supply Valve continues to open until the water flow into the tank equals the flow out of the tank at which point the water level stops falling.
- The system has now reached steady state but the tank level is now lower than the required level (DV>MV) and there is an Offset.
- In a Proportional only system under load there will always be an Offset and that offset will vary dependant on the size of the load.

Proportional Action Summary

- Proportional control will always result in an Offset between Measured Value and Desired Value and for every load there will be a different steady state water level.
- As the Gain increases the Offset decreases.
- As the Gain increases the stability decreases until the system becomes unstable.
- With Proportional only control a compromise must be reached between size of Offset and stability by adjusting the Gain.
- In some systems an Offset is acceptable, as in the water tank described above, and Proportional only control is acceptable.
- However in other systems an offset of any size is unacceptable and some other form of control is required.

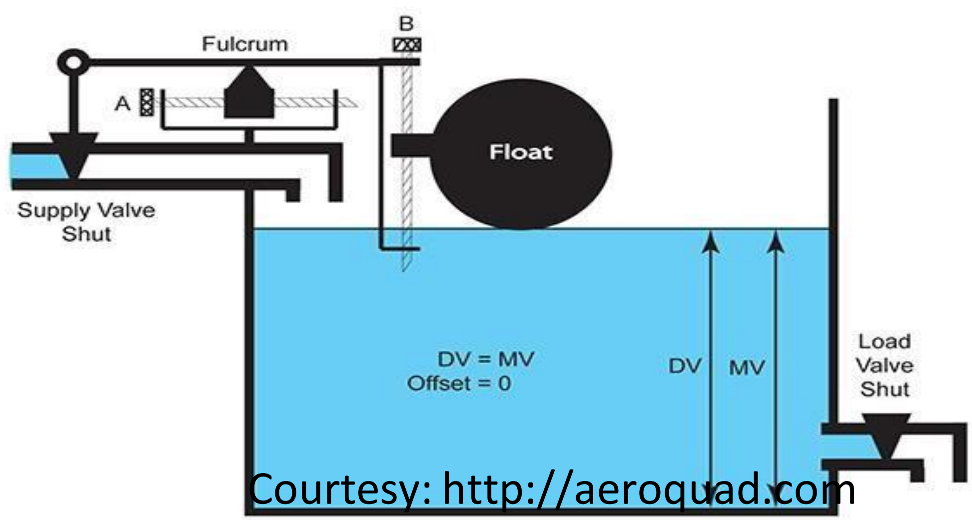
Integral control action

- With the system we described above, under load, assuming the Gain of system has been adjusted to its optimum value, the water level will settle with an Offset from the Desired Value.
- By adjusting knob B so that the float moves upwards, relative to the water level, the Supply Valve will open more, the flow in will increase and the Offset will reduce.
- Eventually a new height of the Float will be found where the flow into the tank equals the flow out , the Measured value equals the Desired Value and Offset will be zero.



Integral control action

- The speed at which the Float height is adjusted can be fast or slow.
- If it is too fast the system can become unstable (hunting) and if it is too slow time will be wasted.
- With Integral control the speed at which Offset is removed is made directly proportional to the size of the Offset.
- In the water tank system, we could achieve this by operating knob B with a variable speed servo motor.
- The amount of integral action applied would be controlled by adjusting the ratio between Motor speed and size of Offset.



Courtesy: http://aeroquad.com

Derivative control action

- Not all systems can be controlled by Proportional and Integral control only.
- In the water tank system, an increase in load results in an immediate drop in water level and the Float.
- The Supply Valve is immediately opened allowing water into the tank.
- In some systems there is a delay or lag in response to a change in load.
- For example, a wind tunnel has a large heavy fan. If more power is applied to increase the fan's speed there will be a significant delay before the new speed is achieved due to the time needed to overcome the inertia of the fan.
- To overcome the inertia more power (than actually required) is required to maintain the desired speed (DV), to accelerate the fans speed change.

Derivative control action

- The additional power is then reduced to the level required to maintain the required speed.
- In the water tank system, under Proportional and Integral control, knob B is operated by a variable speed servo motor.
- If there was inertia in the system, due to friction in the linkage between the Float and the Supply Valve, Derivative action would temporarily apply a higher speed to the servo motor than was necessary to remove the Offset.

Summary:

1. Derivative action speeds up the removal of the Offset.
2. It is required in systems which have large time delays due to Inertia or large capacities.
3. It tends to make a system more stable as it is increased it can cause hunting and instability

P+I+D controllers summary

Proportional action (P)	Arrests	It arrest the change of the Measured Value but always with an Offset from the Measured Value
Integral action (I)	Restores	It removes the Offset
Derivative action (D)	Accelerates	It speeds up the removal of the Offset

References:

1. Modern Control Engineering, Katsuhiko Ogatta, Pearson Education,2004.
2. Control System Engineering, U.A.Bakshi
3. Control Systems, W. Bolton, Elsevier Ltd.
4. http://aeroquad.com/showwiki.php?title=A-Guide-To-Proportional-Integral-and-Derivative-PID-Control

STABILITY ANALYSIS

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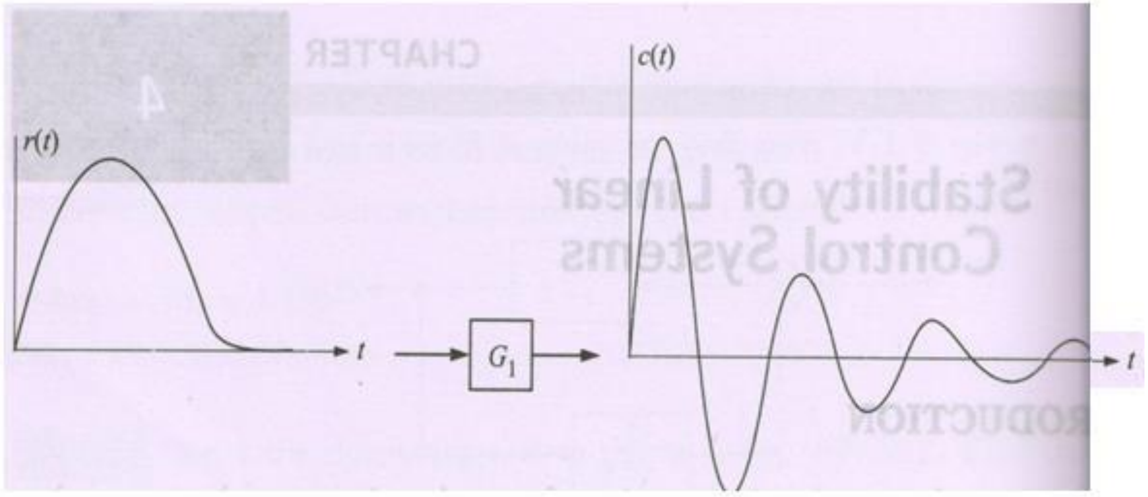
STABILITY ANALYSIS

The analysis of, whether the given system can reach steady state; passing through the transients successfully is called **Stability Analysis** of the system.

A Bounded-Input and a Bounded-Output (BIBO)

A **system** is **stable** if its **impulse response approaches zero as time approaches infinity**

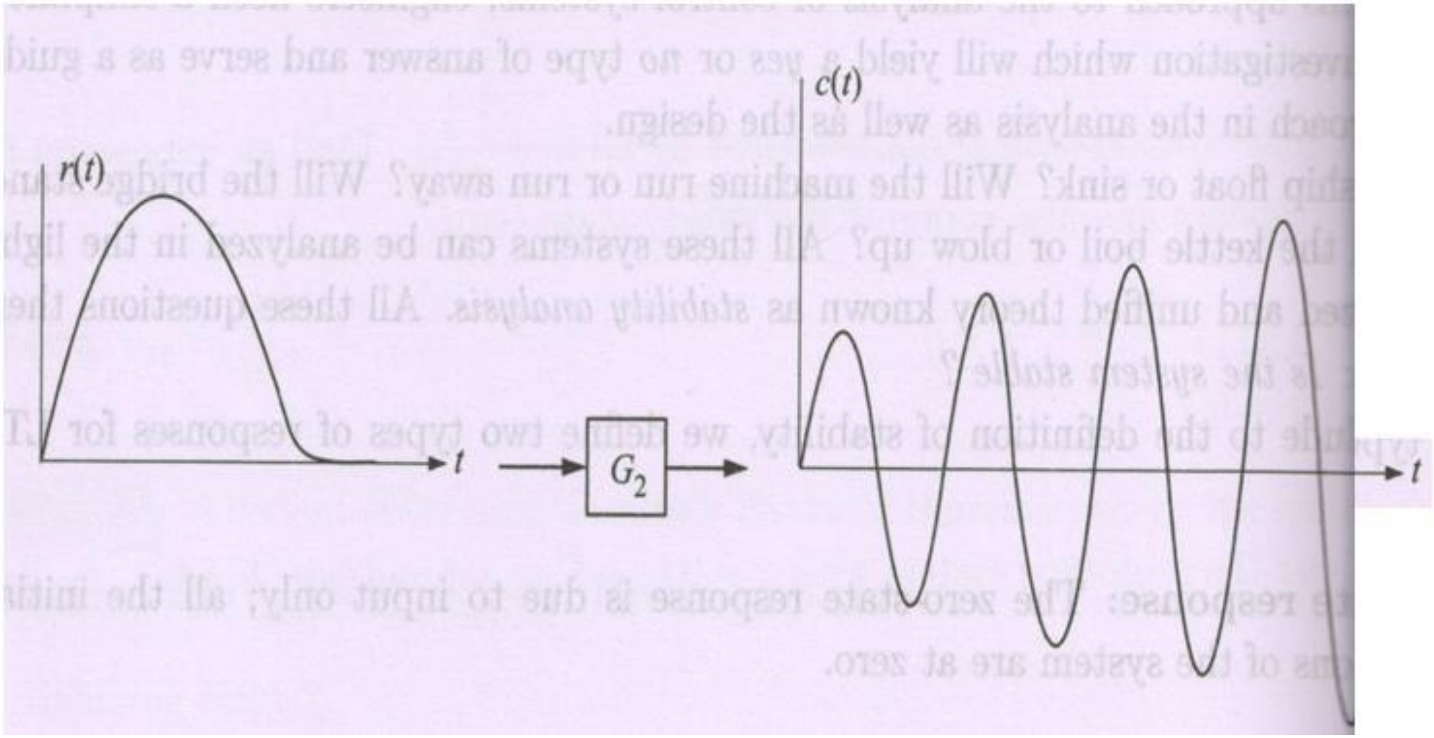
A linear system is said to be stable, if and only if, any bounded input, produces a bounded output.

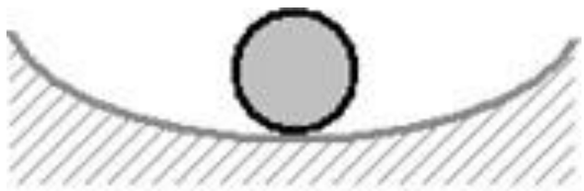


A Bounded-Input and an Unbounded-Output

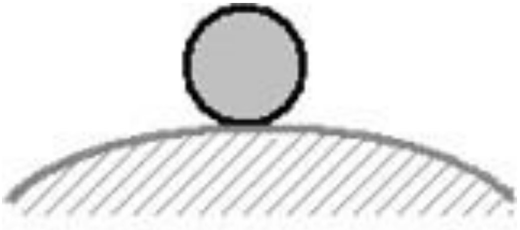
Figure shows a system to which a pulse function has been applied, but the response is seen to be unbounded, therefore the system is unstable.

(A **system** is un**stable** if its **impulse response do not approaches zero as time approaches infinity**)

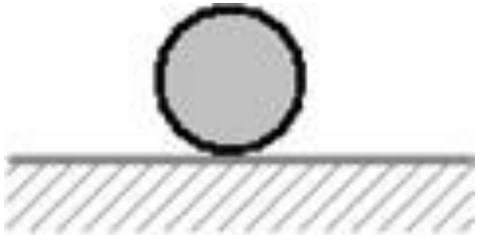




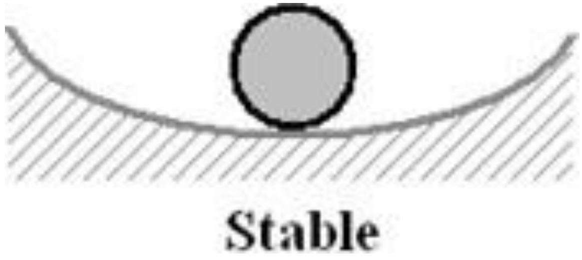
Stable



Unstable



Critical stable



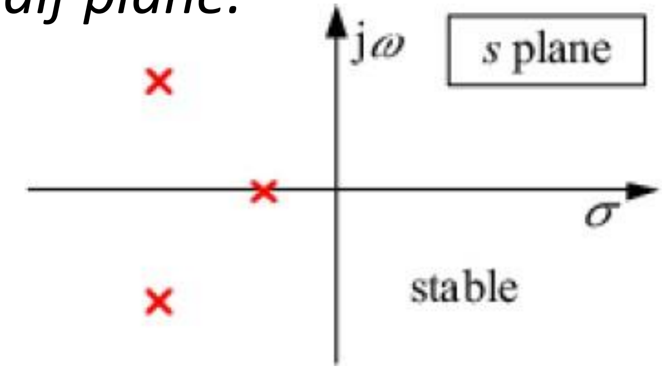
STABLE SYSTEM:

A linear time invariant [LTI]system is said to be **stable** if following conditions are satisfied.

1. When system is excited by a bounded (limited) input, output is also bounded & controllable.
2. In the absence of input, output must tend to zero irrespective of the initial conditions.

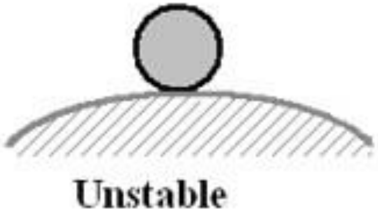
A linear system is stable

If *all poles of its transfer function lie in the left-half plane.*



UNSTABLE SYSTEM:

A linear time invariant [LTI]system is said to be **unstable** if,



1. For a bounded (limited) input it produces unbounded (limitless) output.

2. In the absence of input, output may not be returning to zero.

   (It shows certain output without input)

A linear system is Unstable

If at least one pole of its TF lies in the right-half of the S-plane fig a. **OR**

If at least one multiple pole is on the imaginary axis of the S-plane fig b.

Fig a

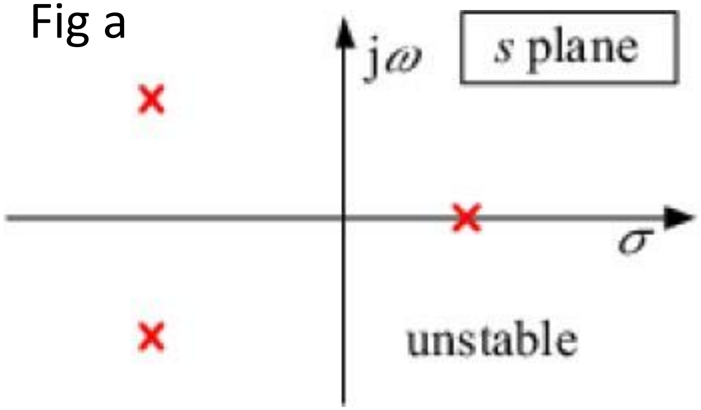
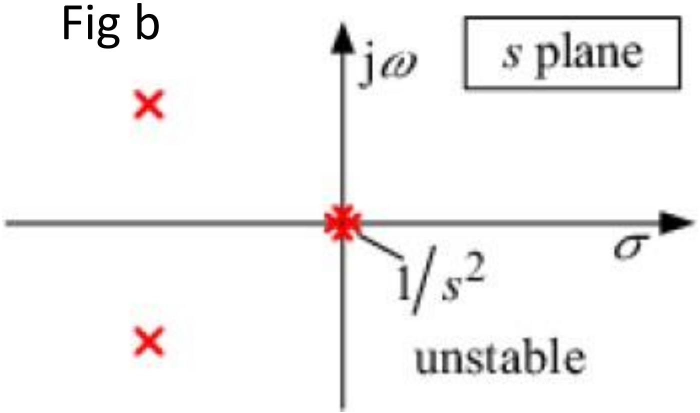


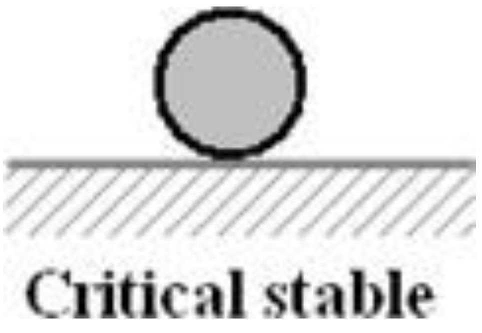
Fig b

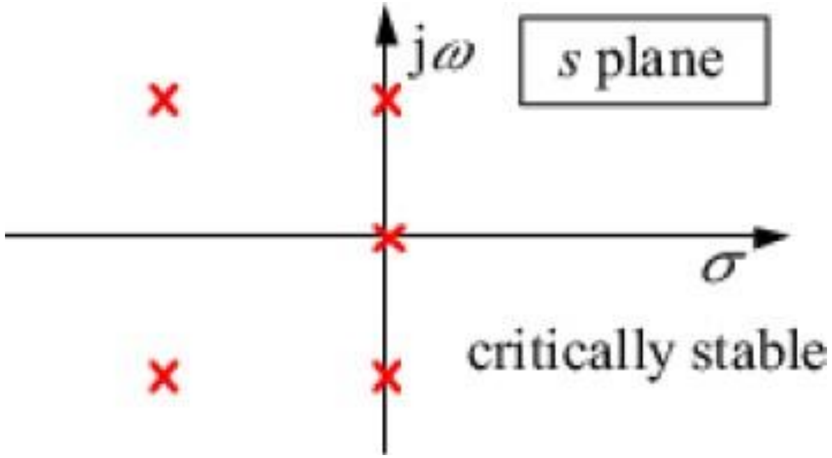


CRITICALLY OR MARGINALLY STABLE SYSTEMS:

A linear system is critically stable,

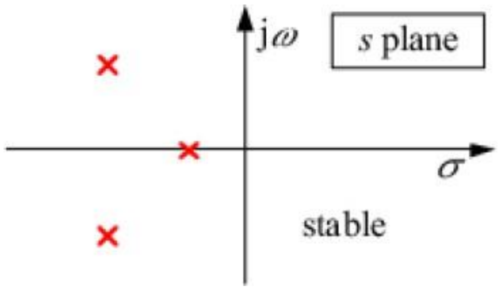
- **if at least one *single pole exists on the imaginary axis,***
- ***no pole of the transfer function lies in the right-half plane***
- ***and in addition no multiple poles lie on the imaginary axis.***





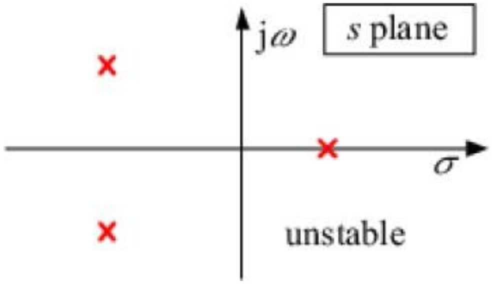
STABLE SYSTEM:

If *all poles of its TF lie in the left-half plane.*



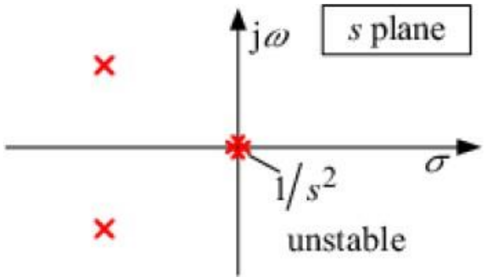
UNSTABLE SYSTEM:

If at least one pole of its TF lies in the right-half of the S-plane



OR

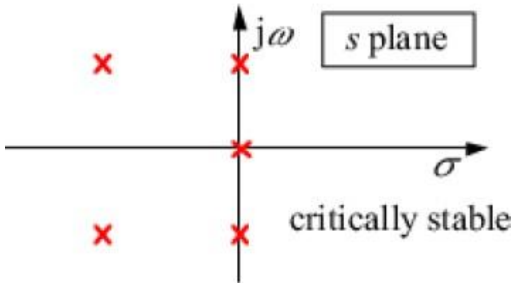
If at least one multiple pole is on the imaginary axis of the S-plane



CRITICALLY STABLE SYSTEMS:

if at least one *single pole exists on the imaginary axis*

- *no pole of the TF lies in the right-half plane*
- *No multiple poles lie on the imaginary axis.*



The different methods available for Stability analysis are

1. Routh – Hurwitz Criterion (Algebraic method in S domain)

2. Graphical methods

    a) Root Locus (Time domain)

    b) Nyquist Plot (Frequency domain)

    c) Bode Plot (Frequency domain)

Routh's Stability Criterion

The necessary condition for system to be stable is —

1. None of the coefficients of the characteristic equation should be missing or zero.
2. All the coefficient should be real and should have the same sign.

These conditions are not sufficient.

The sufficient condition for a system to be stable is that

- Each entry of the first column of Routh's array be positive
- If there are any sign changes existing then
    a) System is Unstable
    b) The number of sign changes equals the number of roots lying in the right half of the S-plane

Construction of Routh's array

Routh suggested a method of tabulating the coefficients of characteristic equation in a particular way.

Tabulation of coefficients gives an array called Routh's array.

Consider the general characteristic equation as,

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots\ldots + a_n = 0.$$

Coefficients of first two rows are written directly from characteristics equation.

Method of forming an array :

$$
\begin{array}{c|cccc}
S^{n} & a_0 & a_2 & a_4 & a_6 \\
S^{n-1} & a_1 & a_3 & a_5 & a_7 \\
S^{n-2} & b_1 & b_2 & b_3 & \\
S^{n-3} & c_1 & c_2 & c_3 & \\
- & - & - & - & \\
- & - & - & - & \\
S^{0} & a_n & & &
\end{array}
$$

From these two rows next rows can be obtained as follows.

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

$$\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & a_6 \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \\
s^{n-2} & b_1 & b_2 & b_3 & \\
s^{n-3} & c_1 & c_2 & c_3 & \\
- & - & - & - & \\
- & - & - & - & \\
s^0 & a_n & & &
\end{array}$$

From 2nd & 3rd row , 4th row can be obtained as

$$C_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}, \qquad C_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

This process is to be continued till the coefficient for s^0 is obtained which will be a_n.

From this array stability of system can be predicted.

Example 1

$$s^3+6s^2 + 11s + 6 = 0$$

Sol: $\mathbf{a_0 = 1, \quad a_1 = 6, \quad a_2 = 11, \quad a_3 = 6}$

S^3	1	11
S^2	6	6
S^1		
S^0		

Example 1

$$s^3+6s^2+11s+6=0$$

Sol: $\mathbf{a_0 = 1, \quad a_1 = 6, \quad a_2 = 11, \quad a_3 = 6}$

$$\begin{array}{c|ccc}
S^3 & 1 & & 11 \\
S^2 & 6 & & 6 \\
S^1 & \dfrac{11 * 6 - 6}{6} & =10 & 0 \\
S^0 & & &
\end{array}$$

Example 1

$$s^3+6s^2+11s+6=0$$

Sol: $\mathbf{a_0 = 1, \quad a_1 = 6, \quad a_2 = 11, \quad a_3 = 6}$

S^3	1	11
S^2	6	6
S^1	$\dfrac{11 * 6 - 6}{6} = 10$	0
S^0	6	

As there is no sign change in the first column, system is stable.

Example 2

$$s^3 + 4s^2 + s + 16 = 0$$

Sol: $a_0 = 1,\quad a_1 = 4,\quad a_2 = 1,\quad a_3 = 16$

S^3	1	1
S^2	+4	16
S^1		
S^0		

Example 2

$$s^3 + 4s^2 + s + 16 = 0$$

Sol: $a_0 = 1, \quad a_1 = 4, \quad a_2 = 1, \quad a_3 = 16$

S^3	1	1
S^2	+4	16
S^1	$\frac{4 - 16}{4} = -3$	0
S^0		

Example 2

$$s^3 + 4s^2 + s + 16 = 0$$

Sol: $a_0 = 1,\quad a_1 = 4,\quad a_2 = 1,\quad a_3 = 16$

$$
\begin{array}{c|ccc}
S^3 & 1 & & 1 \\
S^2 & +4 & & 16 \\
S^1 & \dfrac{4 - 16}{4} & = -3 & 0 \\
S^0 & +16 & &
\end{array}
$$

As there are two sign changes, system is unstable.

Number of roots located in the right half of s-plane = number of sign changes = 2.

Difficulties in RH Criterion

1. When the first term in any row of the Routh's array is zero while rest of the row has atleast one nonzero term.

Methods used to overcome this difficulty:

i. Substitute a small positive number "**ε**" for the zero and proceed.
   Evaluate rest of the Routh array then examine signs of 1st column elements letting $lim_{s \to 0}$.

ii. Modify the original characteristic equation by replacing s by 1/Z. Apply the Routh's test on the modified equation.
   This method works in most but not all cases.

Example $s^3 + 0s^2 + s + 16 = 0$

Sol: $a_0 = 1,\quad a_1 = 0,\quad a_2 = 1,\quad a_3 = 16$

Sol:
$$\begin{array}{c|cc} S^3 & 1 & 1 \\ S^2 & 0 & 16 \\ S^1 & & \\ S^0 & & \end{array}$$

Sol:
$$\begin{array}{c|cc} S^3 & 1 & 1 \\ S^2 & \varepsilon & 16 \\ S^1 & & \\ S^0 & & \end{array}$$

Sol:
$$\begin{array}{c|cc} S^3 & 1 & 1 \\ S^2 & \varepsilon & 16 \\ S^1 & \frac{\varepsilon - 16}{\varepsilon} & \\ S^0 & & \end{array}$$

Sol:
$$\begin{array}{c|cc} S^3 & 1 & 1 \\ S^2 & \varepsilon & 16 \\ S^1 & lim_{\varepsilon\to 0}\frac{\varepsilon - 16}{\varepsilon} & \\ S^0 & & \end{array}$$

Difficulties in RH Criterion

2. When all the elements in any row of the Routh array are zero.

Methods used to overcome this difficulty:

i. Form an equation by using coefficients of a row which is just above the zero row. This equation is called as "Auxillary Equation [A.E]" denoted as A(s).

- ✓ Take derivative of A.E w.r.t 's'
- ✓ Replace the row of zeros by the coefficients of derivatives of A(s) i.e., dA(s)/dt.

Example $S^4+2S^2+1=0$

$S^4+0S^3+2S^2+0S+1=0$

Sol: $a_1=1,\quad a_2=0,\quad a_3=2,\quad a_4=0,\quad a_5=1$

Sol:

S^4	1	2	1
S^3	0	0	
S^2			
S^1			
S^0			

$A(S) = S^4+2\,S^2+1=0$

$$\frac{d\,A(S)}{dt} = 4S^3 + 4S$$

Problem 1

$S^4+S^3+2S^2+10S+8=0$

Sol: $a_1=1$, $a_2=1$, $a_3=2$, $a_4=10$, $a_5=8$

$$
\begin{array}{c|ccc}
S^4 & 1 & 2 & 8 \\
S^3 & 1 & 10 & \\
S^2 & & & \\
S^1 & & & \\
S^0 & & &
\end{array}
$$

Problem 1

$S^4+S^3+2S^2+10S+8=0$

Sol: $a_1=1,\quad a_2=1,\quad a_3=2,\quad a_4=10,\quad a_5=8$

S^4	1	2	8
S^3	1	10	0
S^2	-8	8	
S^1			
S^0			

$$\frac{2-10}{1} = -8$$

$$\frac{8-0}{1} = 8$$

Problem 1

$$S^4+S^3+2S^2+10S+8=0$$

Sol: $a_1=1,\quad a_2=1,\quad a_3=2,\quad a_4=10,\quad a_5=8$

S^4	1	2	8
S^3	1	10	0
S^2	-8	8	0
S^1	11	0	
S^0	8		

$$\frac{-80-8}{-8} = 11$$

$$\frac{0-0}{-8} = 0$$

$$\frac{88-0}{11} = 8$$

Since There are Two sign changes in the first column of Routh's Array, the systems is unstable.
Two roots are present in the right half of the S-plane.

Problem 2

$$S^5+S^4+2S^3+S+5=0$$

Practice at Home ☺

Problem 3 $S^5+2S^4+3S^3+6S^2+2S+1=0$

Sol: $a_1=1$, $a_2=2$, $a_3=3$, $a_4=6$, $a_5=2$, $a_6=1$

s^5	1	3	2
s^4	2	6	1
s^3	0	1.5	
s^2			
s^1			
s^0			

$$\frac{6-6}{2} = 0$$

$$\frac{4-1}{2} = 1.5$$

Problem 3 $S^5+2S^4+3S^3+6S^2+2S+1=0$

Sol: $a_1=1$, $a_2=2$, $a_3=3$, $a_4=6$, $a_5=2$, $a_6=1$

s^5	1	3	2
s^4	2	6	1
s^3	ε	1.5	0
s^2	$\frac{6\varepsilon-3}{\varepsilon}$	1	
s^1			
s^0			

$$\frac{\varepsilon-0}{\varepsilon}=1$$

Problem 3 $S^5+2S^4+3S^3+6S^2+2S+1=0$

Sol: $a_1=1$, $a_2=2$, $a_3=3$, $a_4=6$, $a_5=2$, $a_6=1$

S^5	1	3	2
S^4	2	6	1
S^3	ε	1.5	0
S^2	$\boxed{\frac{6\varepsilon-3}{\varepsilon}}$	1	0
S^1	$\frac{9\varepsilon-4.5-\varepsilon^2}{6\varepsilon-3}$	0	
S^0	1		

$$\frac{1.5\left[\frac{6\varepsilon-3}{\varepsilon}\right]-\varepsilon}{\left[\frac{6\varepsilon-3}{\varepsilon}\right]}$$

$$\frac{9\varepsilon-4.5-\varepsilon^2}{6\varepsilon-3}$$

Problem 3

$S^5+2S^4+3S^3+6S^2+2S+1=0$

Sol: $a_1=1$, $a_2=2$, $a_3=3$, $a_4=6$, $a_5=2$, $a_6=1$

S^5	1	3	2
S^4	2	6	1
S^3	ε	1.5	0
S^2	$\frac{6\varepsilon-3}{\varepsilon}$ → -∞	1	0
S^1	$\frac{9\varepsilon-4.5-\varepsilon^2}{6\varepsilon-3}$ → 1.5	0	
S^0	1		

$$\lim_{\varepsilon\to 0}\left[\frac{6\varepsilon-3}{\varepsilon}\right]$$

$$\lim_{\varepsilon\to 0}\left[6-\frac{3}{\varepsilon}\right]$$

=-∞

$$\lim_{\varepsilon\to 0}\left[\frac{9\varepsilon-4.5-\varepsilon^2}{6\varepsilon-3}\right]$$

$$=\frac{-4.5}{-3}$$

=1.5

Since there are 2 sign changes in the first column of the Routh's Array, system is unstable. Two roots are present in right half of the S-plane

Problem 4

$$S^5+S^4+2S^3+2S^2+3S+15=0$$

Sol: $a_1=1$, $a_2=1$, $a_3=2$, $a_4=2$, $a_5=3$, $a_6=15$

s^5	1	2	3
s^4	1	2	15
s^3	0	-12	0
s^2			
s^1			
s^0			

$$\frac{2-2}{1} = 0$$

$$\frac{3-15}{1} = -12$$

Problem 4 $S^5+S^4+2S^3+2S^2+3S+15=0$

Sol: $a_1=1$, $a_2=1$, $a_3=2$, $a_4=2$, $a_5=3$, $a_6=15$

S^5	1	2	3
S^4	1	2	15
S^3	ε	-12	0
S^2	$\frac{2\varepsilon+12}{\varepsilon}$	15	0
S^1	$\frac{-15\varepsilon^2-24\varepsilon-144}{2\varepsilon+12}$	0	
S^0	15		

$$\frac{15\varepsilon-0}{\varepsilon}=15$$

$$\frac{(-12)\left[\frac{2\varepsilon+12}{\varepsilon}\right]-15\varepsilon}{\left[\frac{2\varepsilon+12}{\varepsilon}\right]}$$

$$=\frac{-15\varepsilon^2-24\varepsilon-144}{2\varepsilon+12}$$

Problem 4

$S^5+S^4+2S^3+2S^2+3S+15=0$

Sol: $a_1=1$, $a_2=1$, $a_3=2$, $a_4=2$, $a_5=3$, $a_6=15$

S^5	1	2	3
S^4	1	2	15
S^3	ε	-12	0
S^2	$\frac{2\varepsilon+12}{\varepsilon}$ (∞)	15	0
S^1	$\frac{-15\varepsilon^2-24\varepsilon-144}{2\varepsilon+12}$ (-12)	0	
S^0	15		

$$\lim_{\varepsilon\to 0}\left[\frac{2\varepsilon+12}{\varepsilon}\right]$$

$= \infty$

$$\lim_{\varepsilon\to 0}\left[\frac{-15\varepsilon^2-24\varepsilon-144}{2\varepsilon+12}\right]$$

$$\frac{-144}{12}$$

$= -12$

Since there are two sign changes, the system is unstable.
Two roots are present in the right half of the S-plane.

Problem 5 $S^6+4S^5+3S^4-16S^2-64S-48=0$

Sol: $a_1=1$, $a_2=4$, $a_3=3$, $a_4=0$, $a_5=-16$, $a_6=64$, $a_7=-48$

S^6	1	3	-16	-48
S^5	$\boxed{4}$	0	-64	0
S^4	3	0	-48	
S^3				
S^2				
S^1				
S^0				

$$\frac{12-0}{4} = 3$$

$$\frac{-64+64}{4} = 0$$

$$\frac{-192-0}{4} = -48$$

Problem 5 $S^6+4S^5+3S^4-16S^2-64S-48=0$

Sol: $a_1=1$, $a_2=4$, $a_3=3$, $a_4=0$, $a_5=-16$, $a_6=64$, $a_7=-48$

S^6	1	3	-16	-48
S^5	4	0	-64	0
S^4	3	0	-48	0
S^3	0	0	0	
S^2				
S^1				
S^0				

$$\frac{-192+192}{3} = 0$$

Problem 5

$S^6+4S^5+3S^4-16S^2-64S-48=0$

Sol: $a_1=1,\quad a_2=4,\quad a_3=3,\quad a_4=0,\quad a_5=-16,\quad a_6=64,\quad a_7=-48$

S^6	1	3	-16	-48
S^5	4	0	-64	0
S^4	3	0	-48	0
S^3	0	0	0	
S^2				
S^1				
S^0				

$A(S) = 3S^4-48=0$

$$\frac{dA(S)}{dt} = 12S^3$$

Problem 5

$S^6+4S^5+3S^4-16S^2-64S-48=0$

Sol: $a_1=1$, $a_2=4$, $a_3=3$, $a_4=0$, $a_5=-16$, $a_6=64$, $a_7=-48$

S^6	1	3	-16	-48
S^5	4	0	-64	0
S^4	3	0	-48	0
S^3	12	0	0	0
S^2	0	-48	0	
S^1				
S^0				

$A(S) = 3S^4-48=0$

$$\frac{dA(S)}{dt} = 12S^3$$

$$\frac{12(-48)-0}{12} = -48$$

Problem 5 $S^6+4S^5+3S^4-16S^2-64S-48=0$

Sol: $a_1=1$, $a_2=4$, $a_3=3$, $a_4=0$, $a_5=-16$, $a_6=64$, $a_7=-48$

S^6	1	3	-16	-48
S^5	4	0	-64	0
S^4	3	0	-48	0
S^3	12	0	0	0
S^2	ε	-48	0	
S^1	$\frac{576}{\varepsilon}$	0		
S^0	-48			

$$\frac{0-(-576)}{\varepsilon} = \frac{576}{\varepsilon}$$

$$\frac{\frac{576}{\varepsilon}(-48)-0}{\frac{576}{\varepsilon}} = -48$$

Problem 5 $S^6+4S^5+3S^4-16S^2-64S-48=0$

Sol: $a_1=1$, $a_2=4$, $a_3=3$, $a_4=0$, $a_5=-16$, $a_6=64$, $a_7=-48$

S^6	1	3	-16	-48
S^5	4	0	-64	0
S^4	3	0	-48	0
S^3	12	0	0	0
S^2	ε	-48	0	
S^1	$\frac{576}{\varepsilon}$	0		
S^0	-48			

$$\lim_{\varepsilon\to 0}\left[\frac{576}{\varepsilon}\right]$$

$= \infty$

$A(S) = 3S^4-48=0$

Let $Y=S^2$

$3Y^2-48=0$

$Y^2=16$; $S^4=16$

$Y= \pm 4 = S^2$

$S^2=+4$ $\quad$ $S^2=-4$

$S=\pm 2$ $\quad$ $S=\pm 2j$

Unstable

Marginal Value K:

The marginal value of ‘K’ is a value which makes any row other than S^0 as row of zeros.

Problem 6 For the system $S^4+22S^3+10S^2+S+K=0$, Find K_{mar} and ω at K_{mar}.

S^4	1	10	K
S^3	22	1	0
S^2	9.95	K	0
S^1	$\frac{9.95-22K}{9.95}$	0	
S^0	K		

$$\frac{220-1}{22} = 9.95$$

$$\frac{22K-0}{22} = K$$

Problem 6 For the system $S^4+22S^3+10S^2+S+K=0$, Find K_{mar} and ω at K_{mar}.

S^4	1	10	K
S^3	22	1	0
S^2	9.95	K	0
S^1	$\boxed{\frac{9.95-22K}{9.95}}$	0	
S^0	K		

The marginal value of K which makes the row of S^1 as row of zeros.

$9.95-22K_{mar}=0$

$K_{mar} = \frac{9.95}{22} = 0.4524$

$A(S) = 9.95S^2+K=0$

Hence, $9.95S^2+0.4524=0$

$S^2= -0.04546$

$S= \pm j\, 0.2132$

$S= \pm j\omega = \pm j\, 0.2132$

$\omega = 0.2132$ rad/sec

Problem 7 For unity feedback system, $G(S)= \dfrac{K}{S(1+0.4S)+(1+0.25S)}$ Find Marginal value of K and frequency of sustained oscillations.

Sol: Characteristic equation is 1+G(S)H(S)=0

$$1+\frac{K}{S(1+0.4S)+(1+0.25S)} = 0$$

$$S[1+0.4S+0.1S^2]+K = 0$$

$$0.1S^3+0.65S^2+S+K = 0$$

S^3	0.1	1
S^2	0.65	K
S^1	$\boxed{\dfrac{0.65-0.1K}{0.65}}$	0
S^0	K	

$$\frac{0.65-0.1K}{0.65} = 0$$

$$0.1K = 0.65$$

$$K=6.5$$

$$A(S) = 0.65S^2+K=0$$

$$0.65S^2+6.5=0$$

$$S^2 = -10$$

$$S = \pm j\,3.162 = \pm j\omega$$

$$\omega = 3.162 \text{ rad/sec}$$

Even there is no sign changes in 1st column of Routh's array, systems is marginally stable because the imaginary roots are present on jw axis.

Problems On Root Locus

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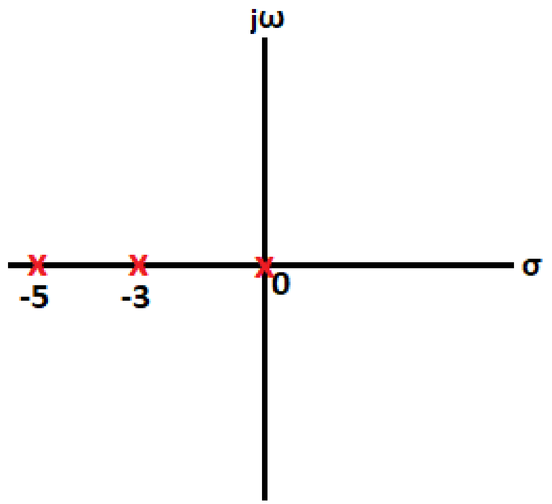
Problem 1:: For $G(S)H(S) = \frac{K}{S(S+3)(S+5)}$ Draw root locus.

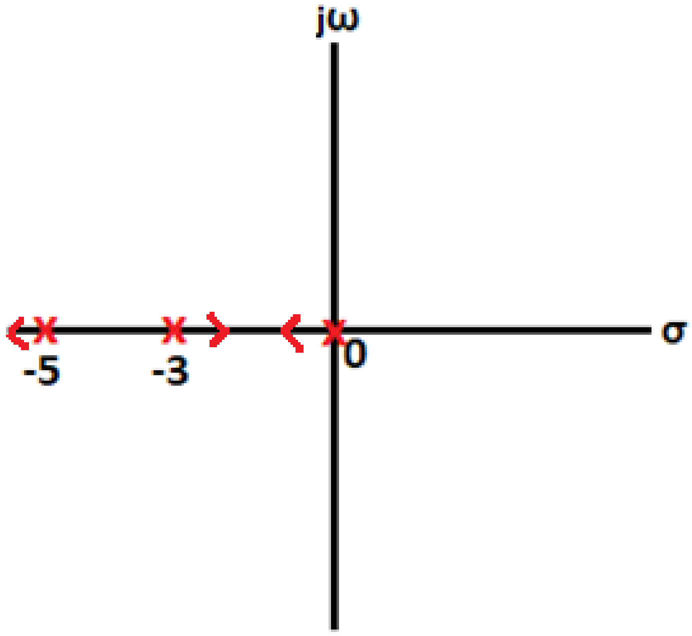
1. Poles=3; S=0,-3,-5; Zeros=0;

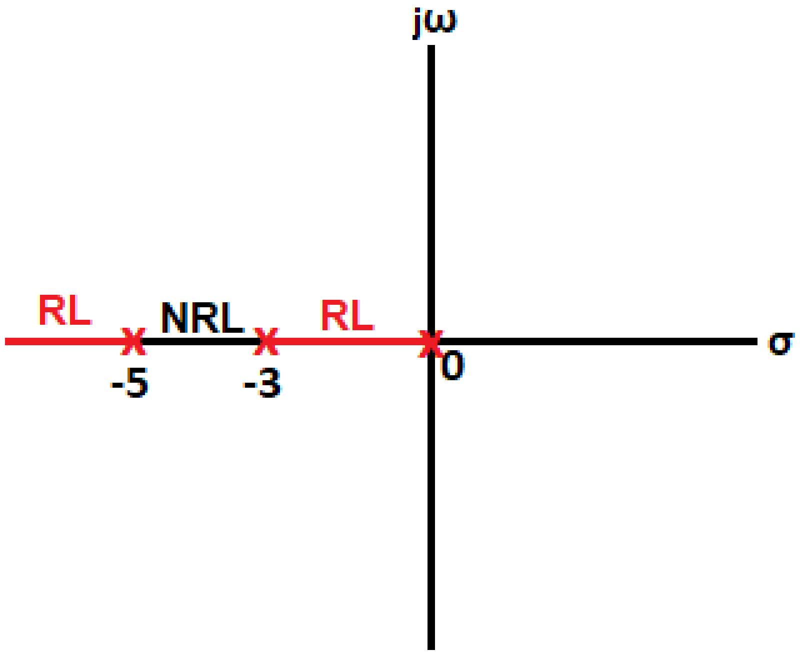
2. Break way points should exist between 0 & -3.

3. No. of Branches of Root Locus (N) = P = 3

4. Identification of Root locus branch.







Problem 1:

5. No. of asymptotes = P-Z = 3-0 = 3

Angle of asymptotes = $\theta = \dfrac{(2q+1)180}{P-Z}$

$\theta 1 = 60^0$ q=0

$\theta 2 = 180^0$ q=1

$\theta 3 = 300^0$ q=2

6. Centroid (σ):

$$\sigma = \frac{\sum \text{Real parts of poles} - \sum \text{Real parts of zeros}}{P-Z}$$

$$\sigma = \frac{(0-3-5)-(0)}{3} = -2.667$$

Problem 1:

7. Break away Point (BAP):

Characteristic equation is 1+G(S)H(S)=0

$$1+\frac{K}{S(S+3)(S+5)}=0$$

$$S^3+8S^2+15S+K=0$$

$$K=-S^3-8S^2-15S \quad \text{(1)}$$

$$\frac{dK}{dS}=-3S^2-16S-15=0$$

$$3S^2+16S+15=0 \quad \text{(2)}$$

Obtain roots for the equation 2. That gives BAP

$S = -1.213$, $S = -4.119$

To check the BAP is valid or not

1. Obtained root must be on Root Locus.
2. Substitute obtained roots "S" in equation 1 and check whether you will get positive value or not.

So, Valid BAP is S = -1.213

Bcz, K= 10.2839 @ S= -1.213

<u>**Problem 1:**</u>

8. Intersection point on imaginary axis:

Characteristic equation is 1+G(S)H(S)=0

$$1+\frac{K}{S(S+3)(S+5)}=0$$

$$S^3+8S^2+15S+K=0$$

$$\begin{array}{c|cc} S^3 & 3 & 15 \\ S^2 & 8 & K \\ S^1 & \frac{120-K}{8} & 0 \\ S^0 & K & \end{array}$$

$$120-K = 0$$

Kmar = 120

Auxillary equation is $A(S) = 8S^2+K = 0$

$$A(S) = 8S^2+120 = 0$$

$$8S^2=-120$$

$$S^2= -15$$

$$S= \pm j3.873$$

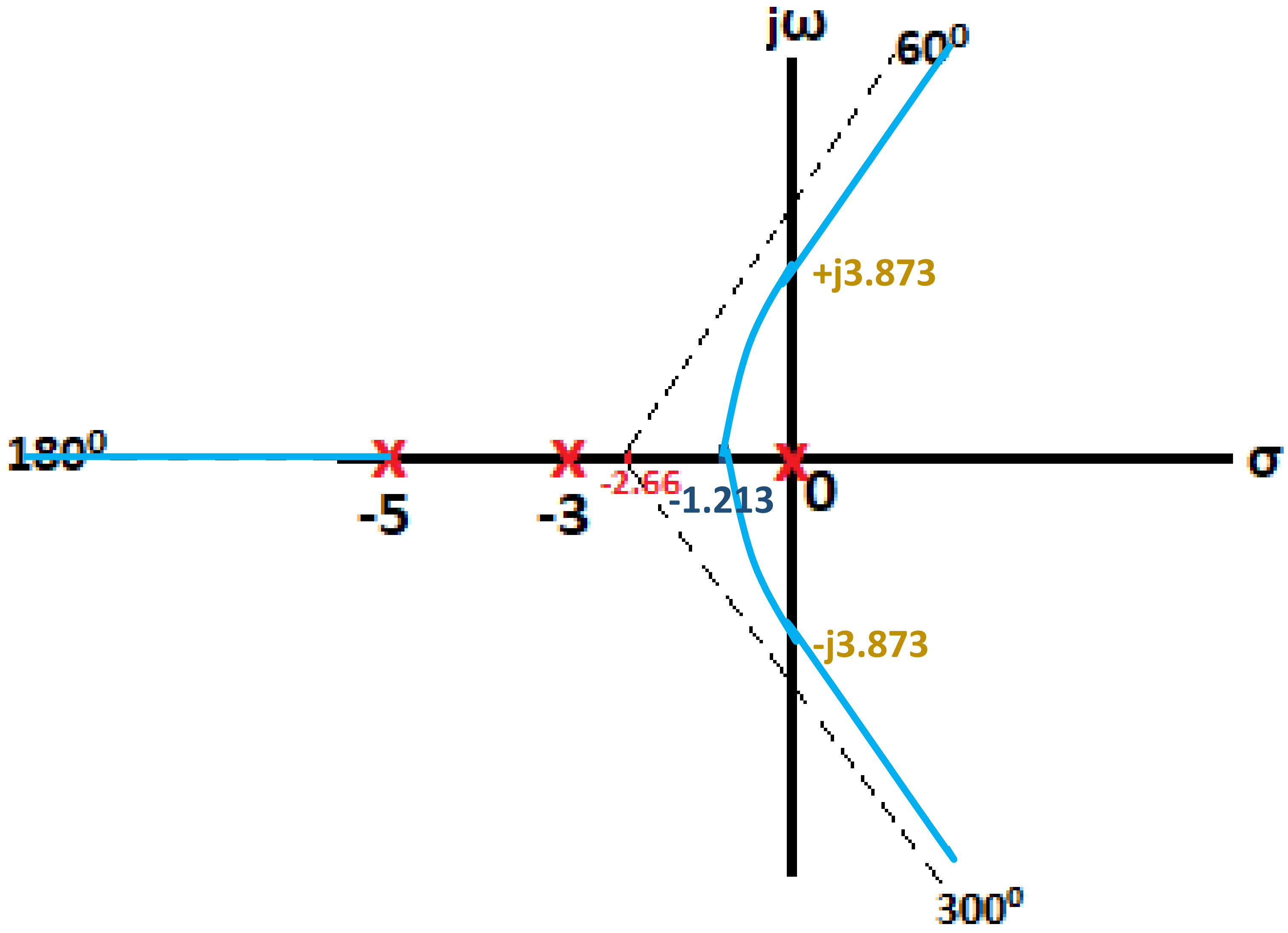
This root is the intersection point on jw axis

Problem 1:

As there are no complex poles and zeros, angle of departure and arrival are not required.

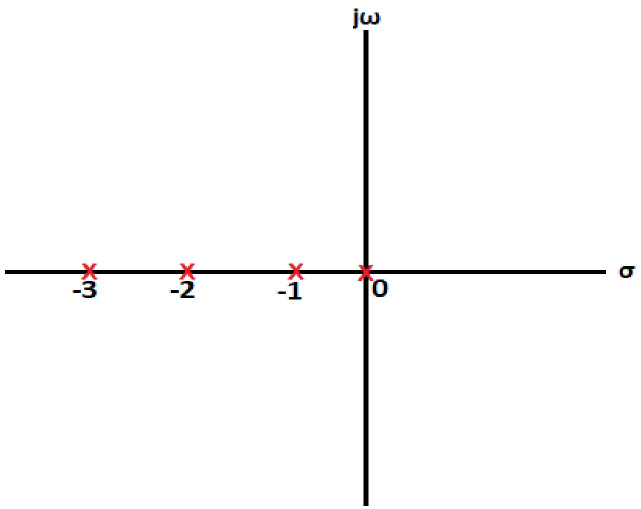
Problem 1:

Root locus Plot



Problem 2: Sketch the complete root locus of the system having

$$G(S)H(S) = \frac{K}{S(S+1)(S+2)(S+3)}.$$

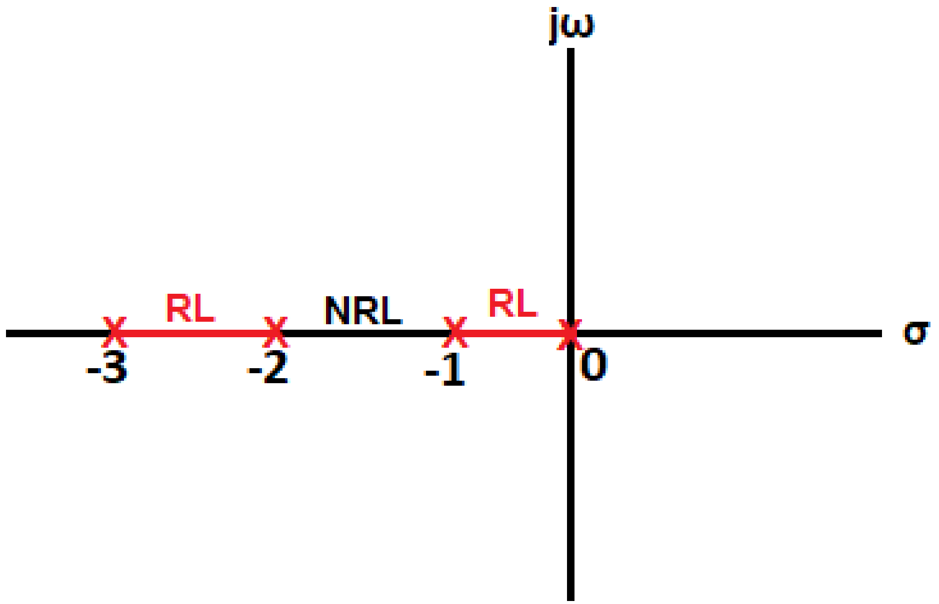


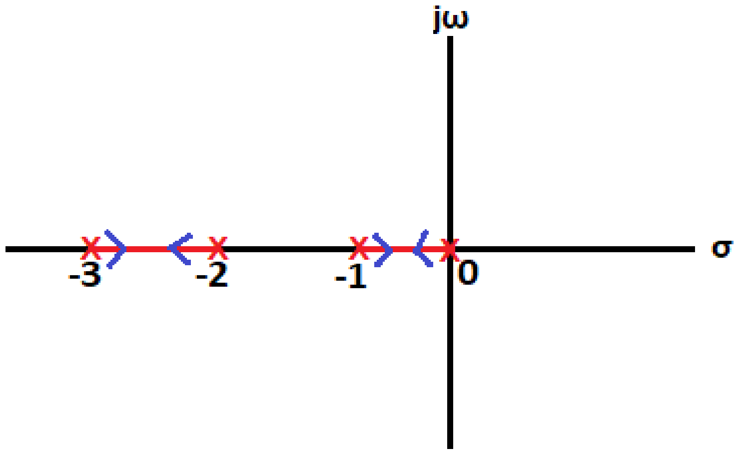
1. Poles=4; S=0,-1,-2,-3; Zeros=0;

2. Break way points should exist between 0 & -1 and -2 & -3.

3. No. of Branches of Root Locus (N) = P = 4

4. Identification of Root locus branch.



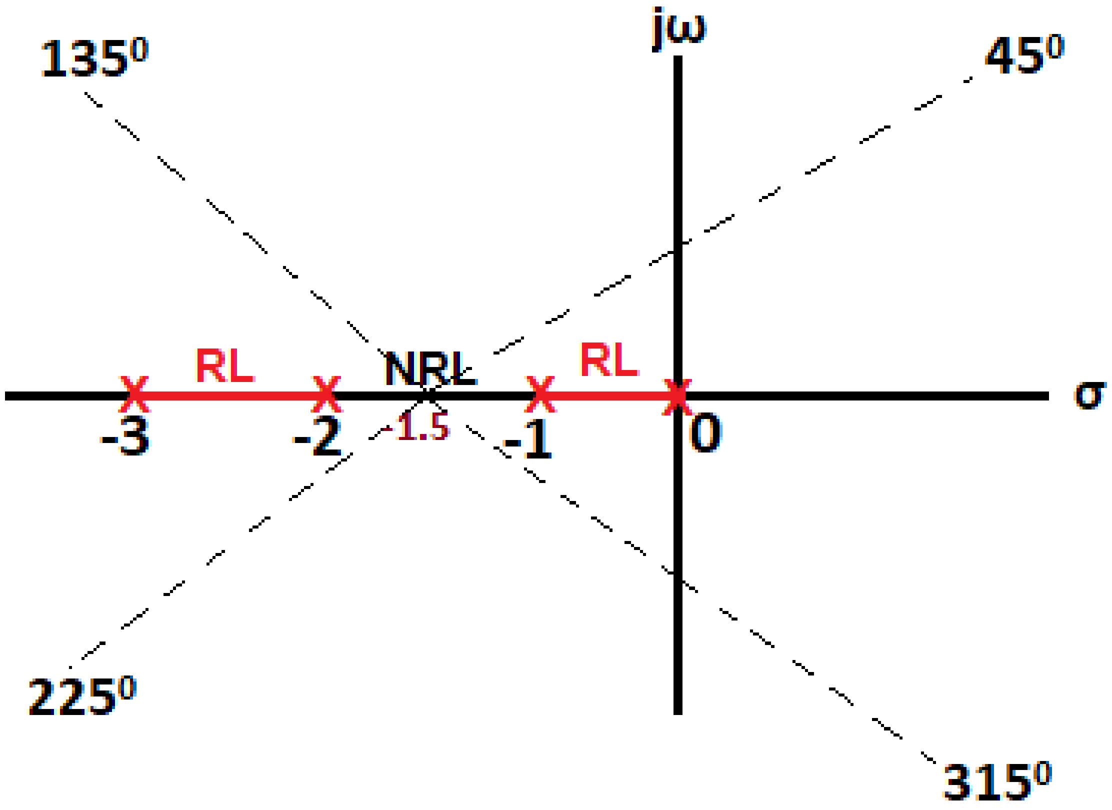


Problem 2:

5. No. of asymptotes = P-Z = 4-0 = 4

Angle of asymptotes = $\theta = \dfrac{(2q+1)180}{P-Z}$

$\theta 1 = 45^0$	q=0
$\theta 2 = 135^0$	q=1
$\theta 3 = 225^0$	q=2
$\theta 4 = 315^0$	q=3



6. Centroid (σ):

$$\sigma = \frac{\sum \text{Real parts of poles} - \sum \text{Real parts of zeros}}{P-Z}$$

$$\sigma = \frac{(0-1-2-3)-(0)}{4} = -1.5$$

Problem 2:

7. Break away Point (BAP):

Characteristic equation is 1+G(S)H(S)=0

$$1+\frac{K}{S(S+1)(S+2)(S+3)}=0$$

$$S^4+6S^3+11S^2+6S+K=0$$

$$K = -S^4-6S^3-11S^2-6S \quad \text{———} \quad (1)$$

$$\frac{dK}{dS} = -4S^3-18S^2-22S-6 = 0$$

$$4S^3+18S^2+22S+6 = 0 \quad \text{———} \quad (2)$$

So, Valid BAP is S = -0.381 & -2.619

Obtain roots for the equation 2. That gives BAP

S = -1.5, S = -0.381, S = -2.619

To check the BAP is valid or not

1. Obtained root must be on Root Locus.
2. Substitute obtained roots "S" in equation 1 and check whether you will get positive value or not.

Problem 2:

8. Intersection point on imaginary axis:

Characteristic equation is $1+G(S)H(S)=0$

$$1+\frac{K}{S(S+1)(S+2)(S+3)}=0$$

$$S^4+6S^3+11S^2+6S+K=0$$

S^4	1	11	K
S^3	6	6	0
S^2	10	K	0
S^1	$\frac{60-6K}{10}$	0	
S^0	K		

$60-6K = 0$

$K_{mar} = 10$

Auxillary equation is $A(S) = 10S^2+K = 0$

$$A(S) = 10S^2+10 = 0$$

$$10S^2=-10$$

$$S^2= -1$$

$$S= \pm j$$

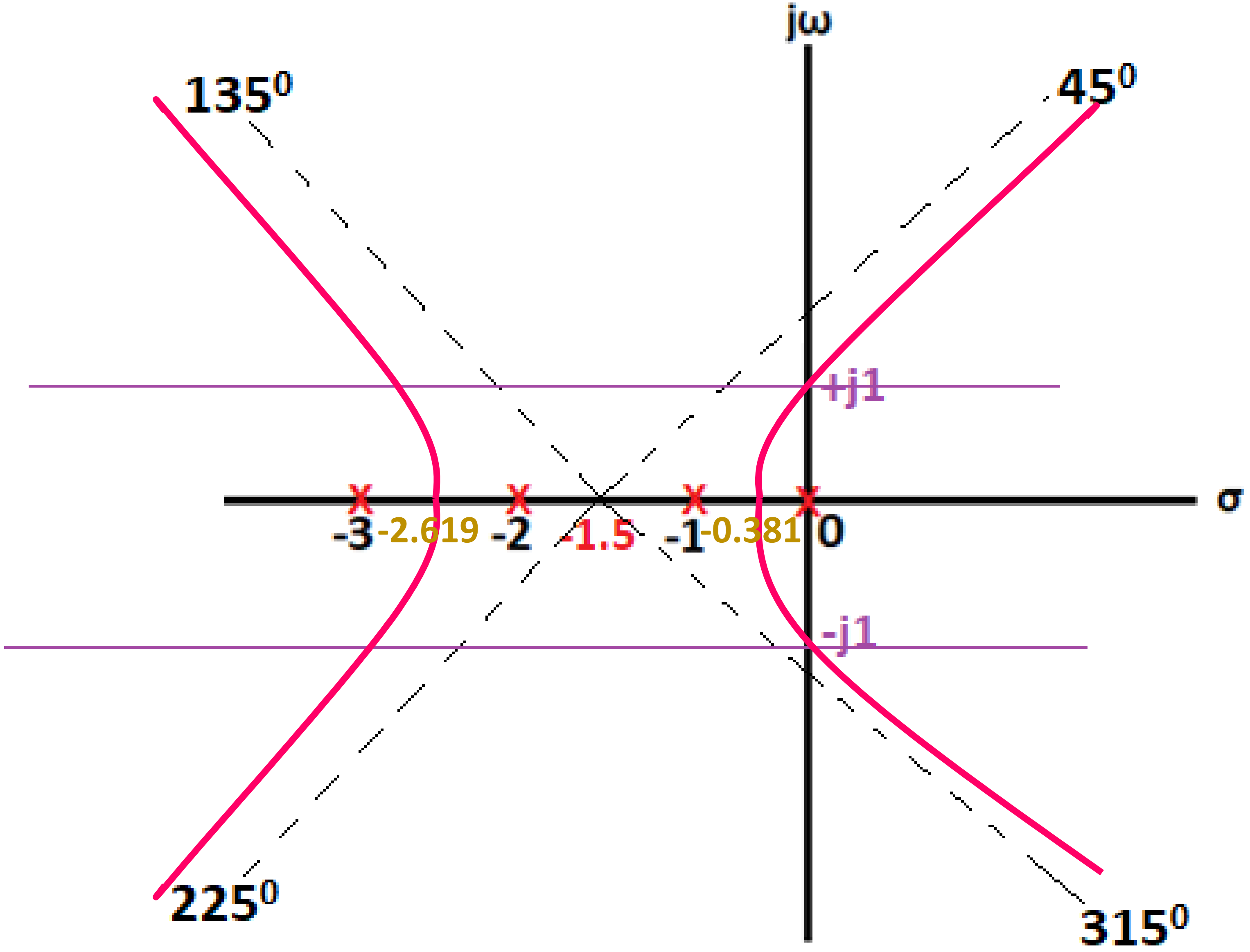
This root is the intersection point on jw axis

Problem 2:

As there are no complex poles and zeros, angle of departure and arrival are not required.

Problem 2:

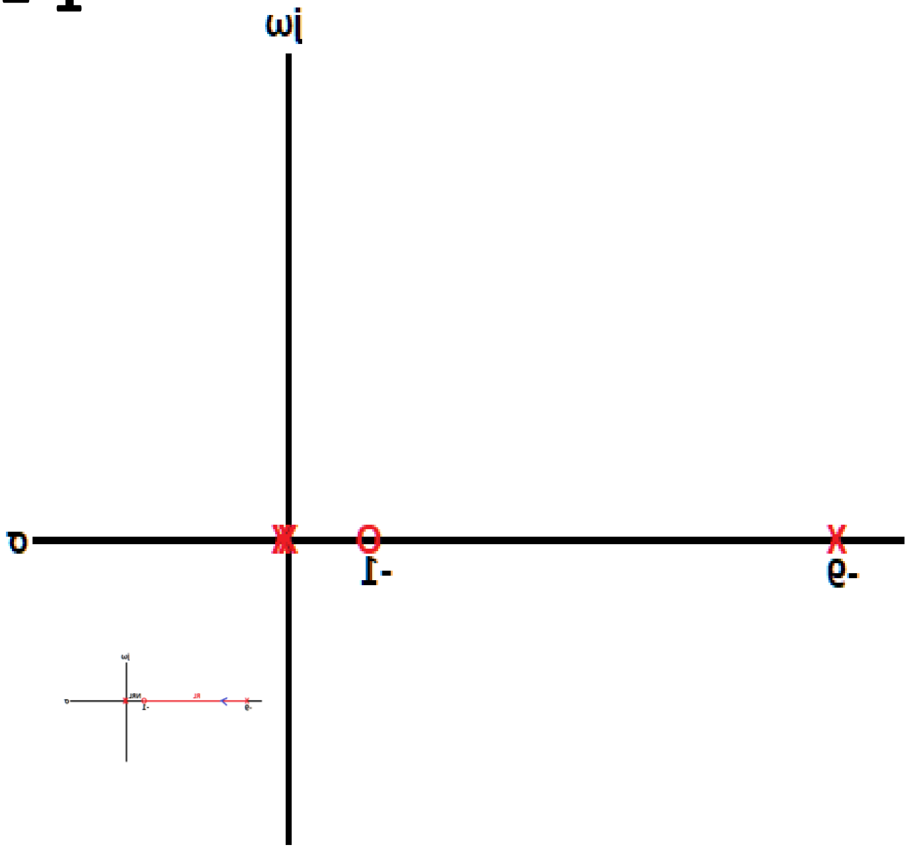
Root locus Plot



Problem 3: Sketch the complete root locus of the system having C.E

$S^3+9S^2+KS+K=0$

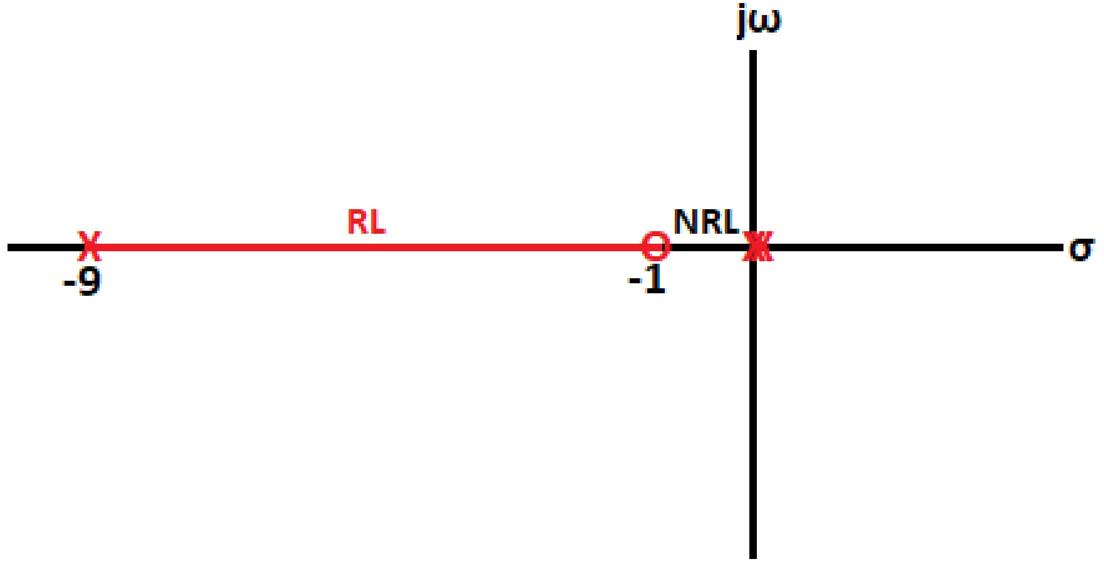
1. Poles=3; S=0,0,-9; Zeros=1; S=-1



2. Break way points should exist between -1 & -9.

3. No. of Branches of Root Locus (N) = P = 3

4. Identification of Root locus branch.



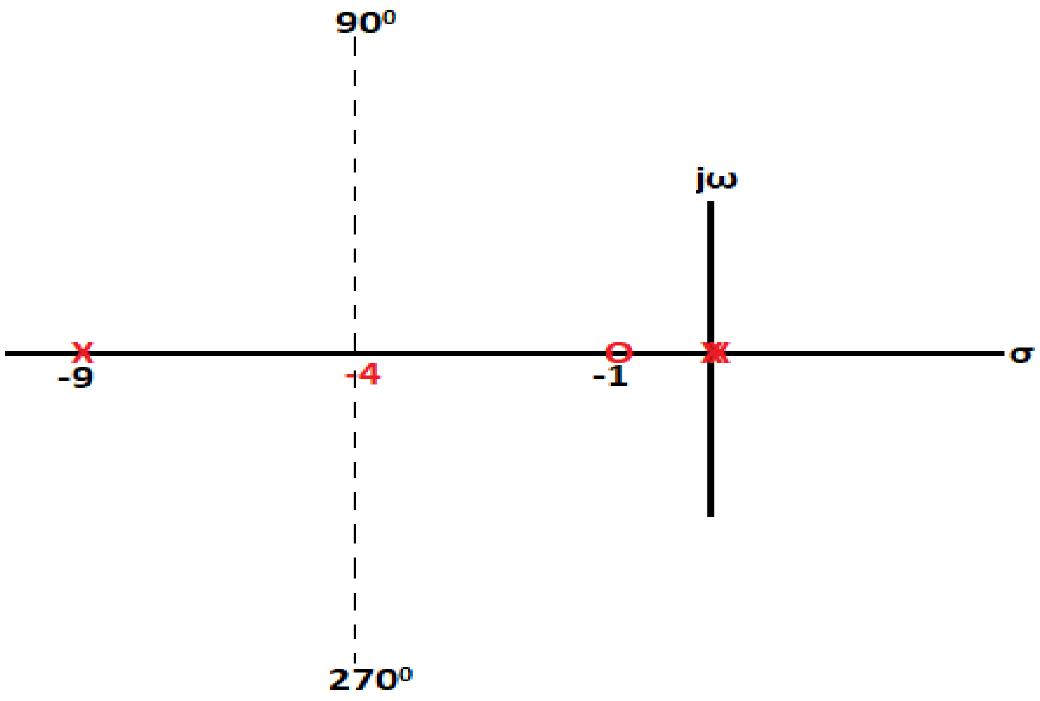
Problem 3:

5. No. of asymptotes = P-Z = 3-1 = 2

Angle of asymptotes = $\theta = \frac{(2q+1)180}{P-Z}$

$\theta 1 = 90^0$ q=0

$\theta 2 = 270^0$ q=1



6. Centroid (σ):

$$\sigma = \frac{\sum \text{Real parts of poles} - \sum \text{Real parts of zeros}}{P-Z}$$

$$\sigma = \frac{(0-0-9)-(-1)}{2} = -4$$

Problem 3:

7. Break away Point (BAP):

Characteristic equation is $1+G(S)H(S)=0$

$$S^3+9S^2+KS+K=0$$

$$K=\frac{-S^3-9S^2}{(S+1)} \quad \text{———— } 1$$

$$\frac{dK}{dS}=\frac{(S+1)(-3S^2-18S)-(-S^3-9S^2)(1)}{(S+1)^2}=0$$

$$-3S^3-3S^2-18S^2-18S+S^3+9S^2=0$$

$$-2S^3-12S^2-18S=0$$

$$S\,(2S^2+12S+18)=0 \quad \text{———— } 2$$

So, Valid BAP is S = -3

Obtain roots for the equation 2. That gives BAP

$S = 0$, $S = -3$, $S = -3$

To check the BAP is valid or not

1. Obtained root must be on Root Locus.
2. Substitute obtained roots "S" in equation 1 and check whether you will get positive value or not.

Problem 3:

8. Intersection point on imaginary axis:

Characteristic equation is 1+G(S)H(S)=0

$$S^3+9S^2+KS+K=0$$

S^3	1	K
S^2	9	K
S^1	$\frac{8K}{9}$	0
S^0	K	

System cant work with K=0.
This indicates that there is no intersection of root locus with imaginary axis.

8K = 0

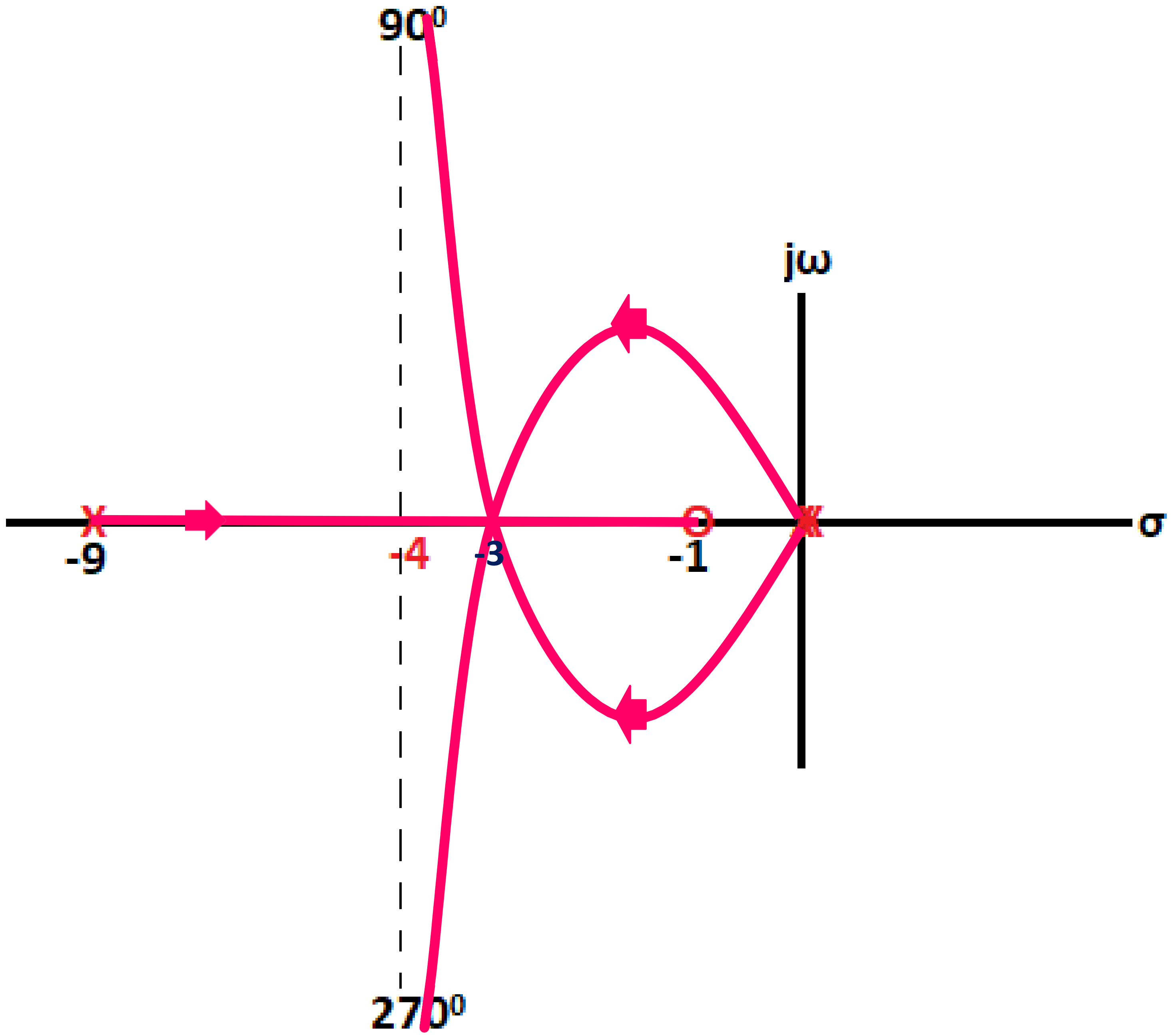
Kmar = 0

Problem 3:

As there are no complex poles and zeros, angle of departure and arrival are not required.

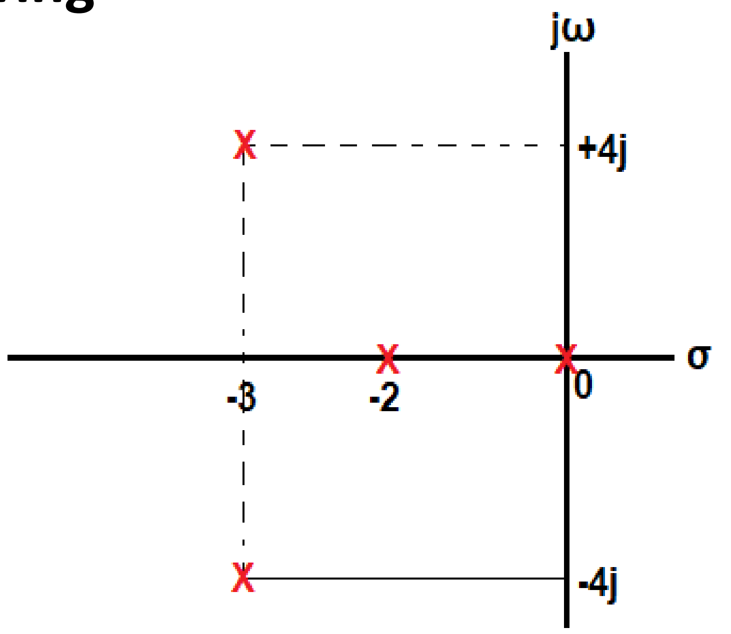
Problem 3:

Root locus Plot



Problem 4: Sketch the complete root locus of the system having

$$G(S) = \frac{K}{S(S+2)(S^2+6S+25)}.$$

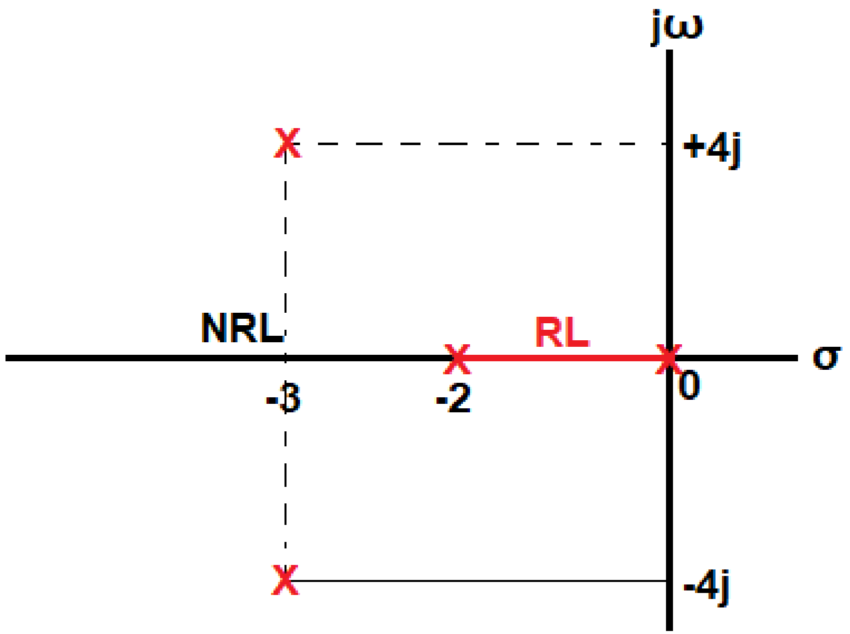


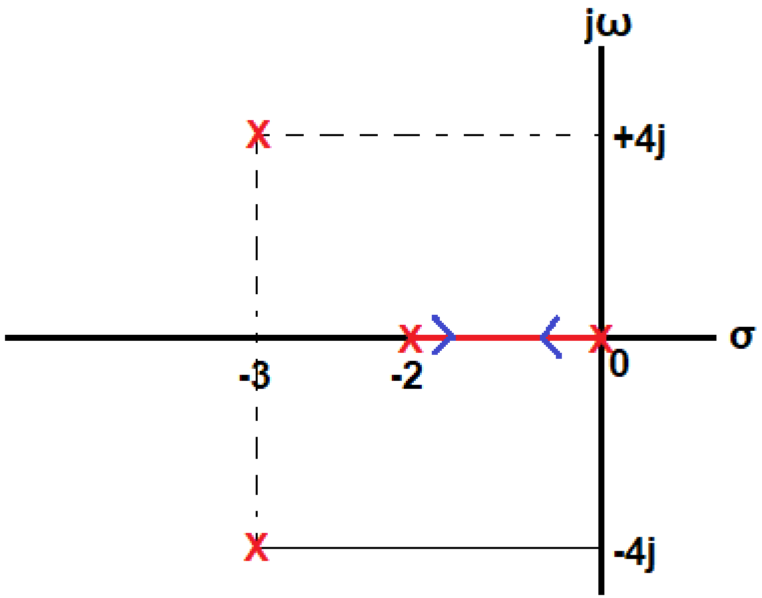
1. Poles=4; S=0, -2, -3+4j, -3-4j; Zeros=0;

2. Break way points should exist between 0 & -2.

3. No. of Branches of Root Locus (N) = P = 4

4. Identification of Root locus branch.



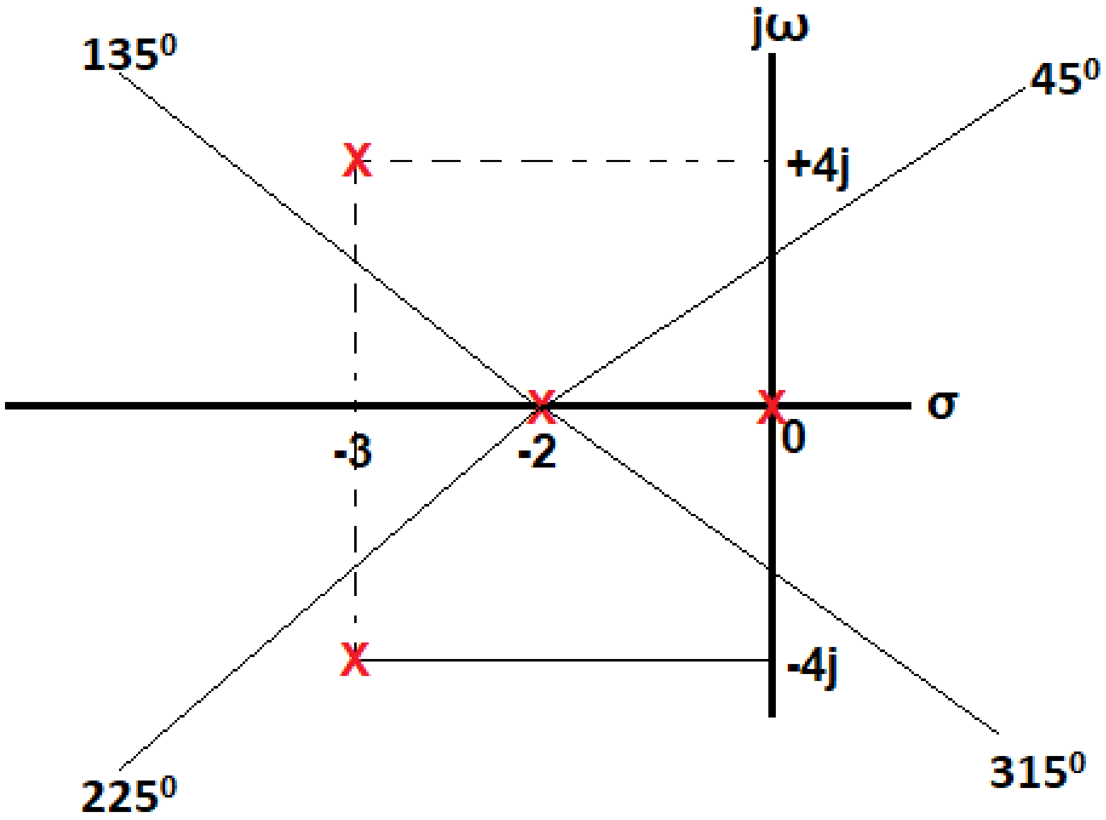


Problem 4:

5. No. of asymptotes = P-Z = 4-0 = 4

Angle of asymptotes = $\Theta = \frac{(2q+1)180}{P-Z}$

$\Theta1 = 45^0$	q=0
$\Theta2 = 135^0$	q=1
$\Theta3 = 225^0$	q=2
$\Theta4 = 315^0$	q=3



6. Centroid (σ):

$$\sigma = \frac{\sum \text{Real parts of poles} - \sum \text{Real parts of zeros}}{P-Z}$$

$$\sigma = \frac{(0-2-3-3)-(0)}{4} = -2$$

Problem 4:

7. Break away Point (BAP):

Characteristic equation is $1+G(S)H(S)=0$

$$S^4+8S^3+37S^2+50S+K=0$$

$$K=-S^4-8S^3-37S^2-50S \qquad (1)$$

$$\frac{dK}{dS}=-4S^3-24S^2-74S-50=0$$

$$4S^3+24S^2+74S+50=0 \qquad (2)$$

So, Valid BAP is $S = -0.898$

Obtain roots for the equation 2. That gives BAP

$S=-0.898$, $\quad S=-2.55\pm2.722j$

To check the BAP is valid or not

1. Obtained root must be on Root Locus.
2. Substitute obtained roots "S" in equation 1 and check whether you will get positive value or not.

Problem 4:

8. Intersection point on imaginary axis:

Characteristic equation is 1+G(S)H(S)=0

$$S^4+8S^3+37S^2+50S+K=0$$

S^4	1	37	K
S^3	8	50	0
S^2	30.75	K	
S^1	$\frac{1537.5-8K}{30.75}$	0	
S^0	K		

$A(S) = 30.75S^2+K=0$

$30.75S^2+ 182.1875=0$

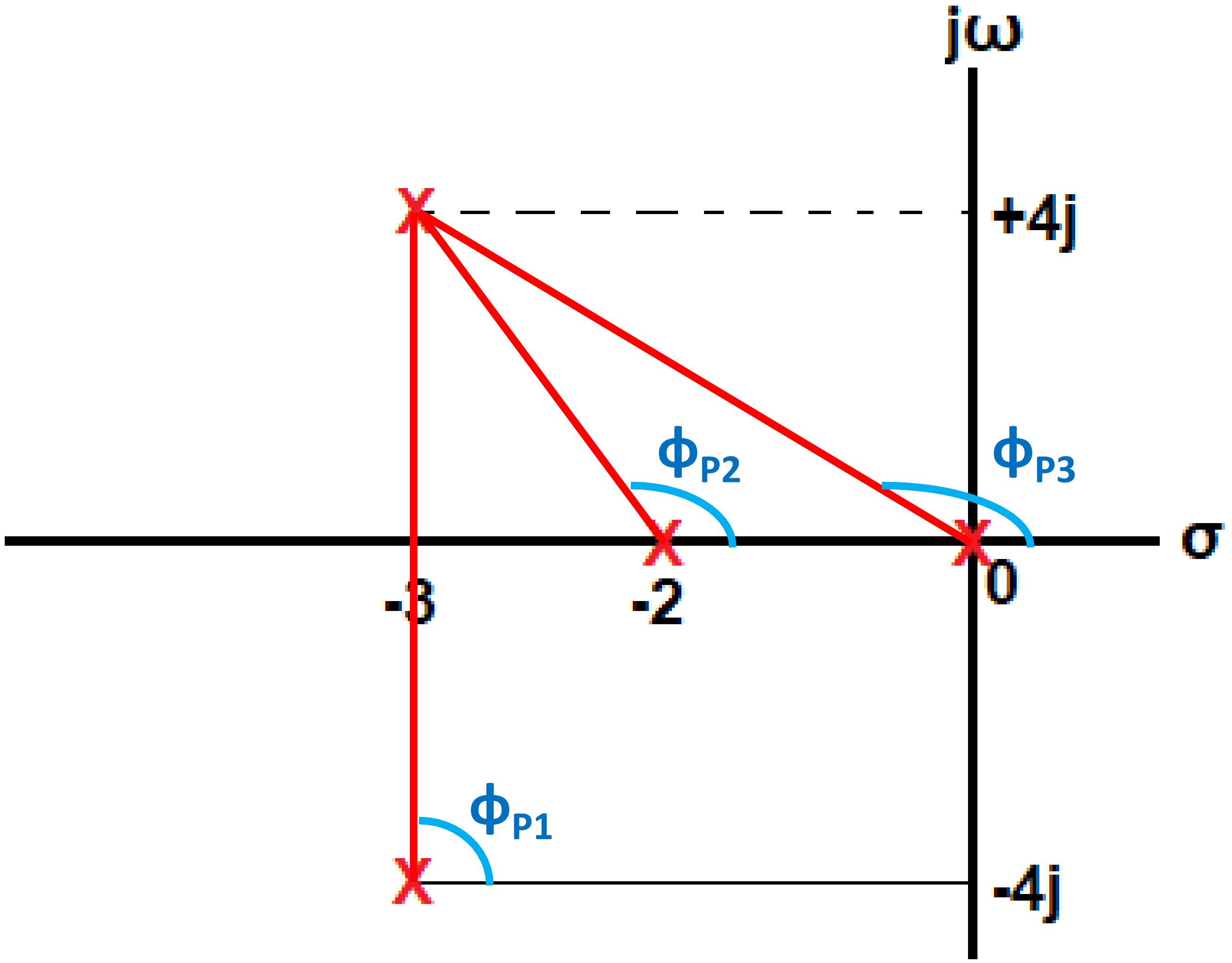
$S= \pm j\ 2.5$ is the intersection point on jw axis.

$1537.5-8K = 0$

Kmar = 182.1875

Problem 4:

9. Angle of departure:



ϕ_{P1} = 90^0

ϕ_{P2} = 180^0 - $\tan^{-1}\left(\frac{4}{1}\right)$

ϕ_{P2} = 101.03^0

ϕ_{P3} = 180^0 - $\tan^{-1}\left(\frac{4}{3}\right)$

ϕ_{P3} = 126.86^0

Angle of departure:

$\phi_d = 180^0 - [\sum \phi_P - \sum \phi_Z]$

$\sum \phi_P = \phi_{P1} + \phi_{P2} + \phi_{P3}$

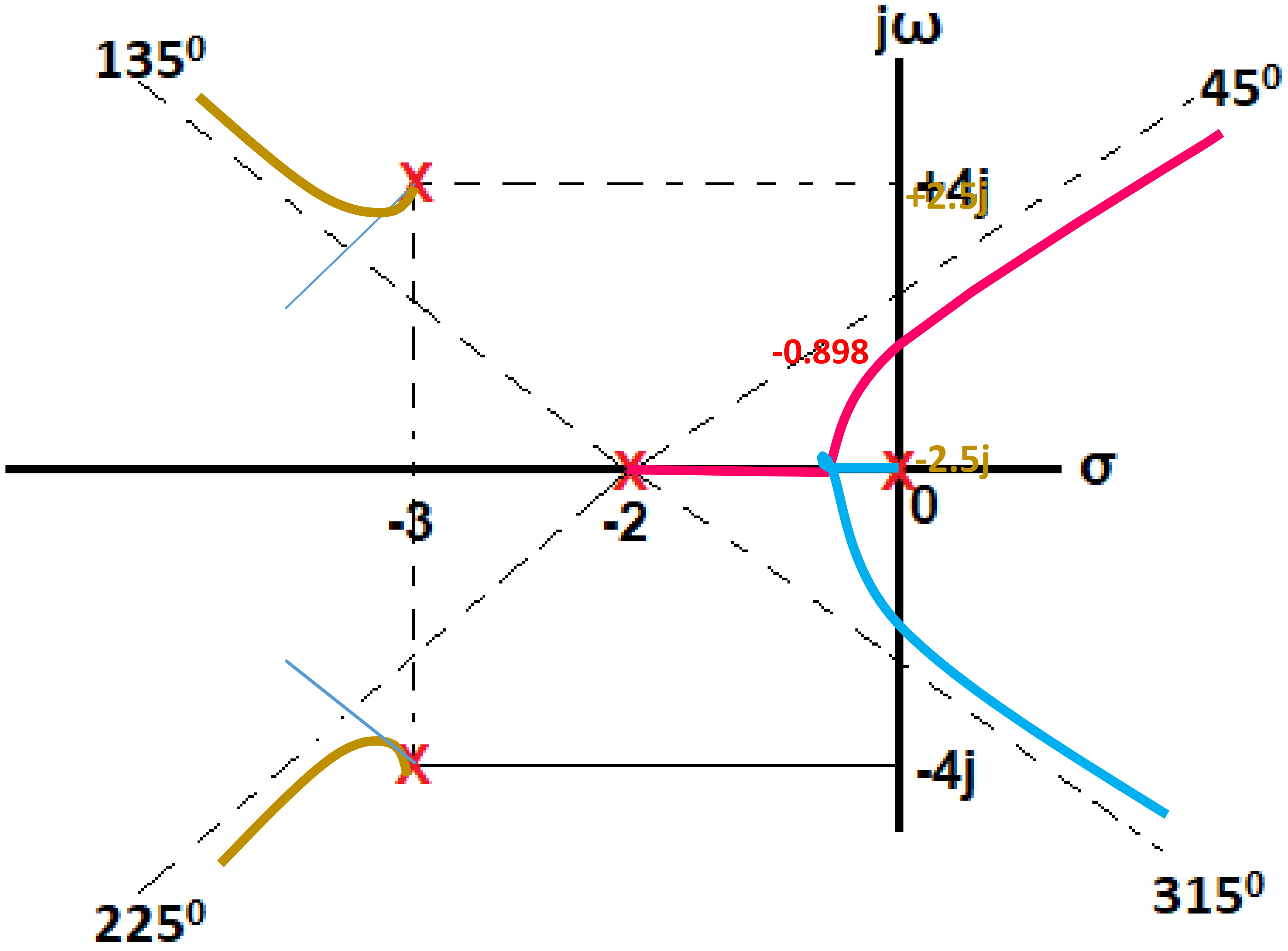
$= 90^0 + 101.03^0 + 126.86^0$

$= 320.89^0$

$\phi_d = 180^0 - 320.89^0 = -140.89^0$

Problem 4:

Root locus Plot



Thank You ☺